Math 110 — Homework Assignment 8

DUE DATE: Nov. 11, 2016

1. Determine if the linear transformations described by the following matrices are invertible. If not, explain why, and if so, find the matrix of the inverse transformation.

(a) $\begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 & 6 \\ 0 & 3 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 7 & 3 \\ 9 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$

(e) $\begin{bmatrix} 3 & 1 & 5 \\ 6 & 3 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ (f) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 5 & 6 & 1 & 0 \\ 7 & 10 & 4 & 1 \end{bmatrix}$

2. Suppose that A is the matrix

$$A = \begin{bmatrix} 5 & 2 & 4 \\ 2 & 3 & 1 \\ 5 & 6 & 3 \end{bmatrix}.$$

(a) Find the inverse of A.

(b) Explain why, for any values of a, b, and c, the equations

$$\begin{array}{ccccccc} 5x & + & 2y & + & 4z & = & a \\ 2x & + & 3y & + & z & = & b \\ 5x & + & 6y & + & 3z & = & c \end{array}$$

always have a unique solution.

(c) Find this unique solution (in terms of a, b, and c).

3. Suppose that $T_1 : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ and $T_2 : \mathbb{R}^m \longrightarrow \mathbb{R}^p$ are linear transformations.

(a) If T_1 and T_2 are injective, prove that $T_2 \circ T_1$ is injective.

(b) If T_1 and T_2 are surjective, prove that $T_2 \circ T_1$ is surjective.

(c) If T_1 and T_2 are invertible, prove that $T_2 \circ T_1$ is invertible.

4. For each of the following subsets W of $\mathbb{R}^3$, either show that they are subspaces, or show why they aren't subspaces by explaining which of the three conditions don't hold.

(a) $W = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 = z^2\}$.

(b) $W = \{(x, y, z) \in \mathbb{R}^3 \mid (x, y, z) \text{ is orthogonal to } (3, 1, -2)\}$.

(c) $W = \{(x, y, z) \in \mathbb{R}^3 \mid x + y + z \geqslant 0\}$.