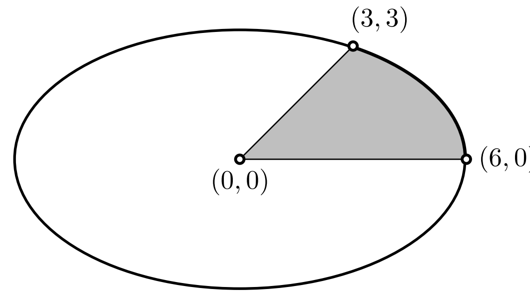
DUE DATE: JAN. 20, 2016

1. The ellipse at right is given by the equation

$$\frac{x^2}{36} + \frac{y^2}{12} = 1.$$

Find the area of the shaded region.



HINT: The ellipse is the image of the unit circle under a linear transformation. First, figure out what that linear transformation is. Then figure out what region in the unit circle is sent to the shaded area under that linear transformation. If you can compute the area of the region in the unit circle, you'll be able to compute the area of its image in the ellipse.

2. Suppose that $T\colon \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is a linear transformation, and that the determinant of T is not zero.

(a) If S_1 and S_2 are two shapes in $\mathbb{R}^2$ (with $\text{Area}(S_2) \neq 0$) explain why

$$\frac{\text{Area}(S_1)}{\text{Area}(S_2)} = \frac{\text{Area}(T(S_1))}{\text{Area}(T(S_2))}$$

i.e., that a linear transformation preserves the ratios of areas.

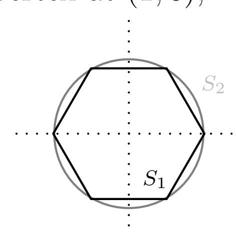
(b) If ℓ is any line in $\mathbb{R}^2$ (not necessarily passing through the origin), explain why $T(\ell)$ is also a line (again, not necessarily passing through the origin).

SUGGESTION: You may find it easier to analyze what happens to ℓ under a linear transformation by thinking of a parameterization of ℓ of the form $p + t\vec{v}$.

Now let T be the linear transformation with standard matrix $\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$.

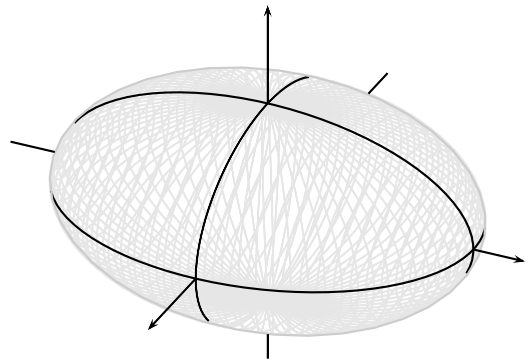
(c) Let S_2 be the unit circle, and S_1 the inscribed hexagon with one vertex at $(1, 0)$, as shown at right.

Draw a sketch of $T(S_2)$ and $T(S_1)$.



(d) Find $\dfrac{\text{Area}(T(S_1))}{\text{Area}(T(S_2))}$.

3. The ellipsoid shown at right has equation



$$\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{4} = 1.$$

Find the volume of the ellipsoid.
(Using linear algebra, of course.)