DUE DATE: FEB. 10, 2017

1. Let A be the matrix

$$A = \begin{bmatrix} 3 & 2 & z \\ x & 1 & 3 \\ 2 & y & 5 \end{bmatrix},$$

where x, y, and z are variables.

(a) Compute the adjoint matrix of A (this will still involve the variables x, y, and z).

(b) Compute the product of the adjoint matrix and A.

(c) Compute $\det(A)$.

(d) Assuming that $\det(A) \neq 0$, write down the inverse of A (this will still be a matrix involving x, y, and z).

(e) To show that this method really gives a "universal formula for the inverse", plug in the values $(x, y, z) = (3, -1, 1)$ into both A and the inverse matrix from part (d), and multiply them to see that it really gives the inverse for A.

(f) Do the same for $(x, y, z) = (1, 1, 1)$

2. Suppose that A is an $n \times n$ matrix with integer entries, and that A is invertible. Since A is invertible, we can compute the matrix A^{-1}. The computations seem much cleaner (and friendlier) when A^{-1} also has only integer entries. The purpose of this question is to figure out when that happens.

(a) Show that if $\det(A) = \pm 1$ then A^{-1} has only integer entries. (SUGGESTION: Use the expression for the inverse in terms of the adjoint matrix.)

(b) Conversely suppose that A^{-1} also has integer entries. Using the fact that

$$\det(A)\det(A^{-1}) = \det(I_n) = 1,$$

explain why we must have $\det(A) = \pm 1$.

(c) Conclude that if A is a square matrix with integer entries then A^{-1} has integer entries if and only if $\det(A) = \pm 1$.

3. Consider the following matrix A, and the transpose of its adjoint :

$$A = \begin{bmatrix} 2 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \\ 1 & 1 & 2 & 2 \end{bmatrix} \quad \text{and} \quad \operatorname{adj}(A)^t = \begin{bmatrix} -3 & 3 & 3 & -3 \\ 0 & -2 & -2 & 3 \\ 3 & -5 & -2 & 3 \\ 0 & 3 & 0 & -3 \end{bmatrix}.$$

(a) Compute $A \cdot \operatorname{adj}(A)$.

(b) What is $|A|$?

Answer the following questions by using the Laplace expansion formula to deduce how the determinant of A changes when we change a single entry. When using the Laplace expansion formula, you will need to know the determinants of certain of 3×3 submatrices of A, but you can read that off from the matrix $\operatorname{adj}(A)^t$ above. (Making this step slightly easier was the reason for writing the transpose of the adjoint above, instead of the adjoint.)

(c) If we add 3 to a_{14}, what is the determinant of the new matrix?

(d) Going back to the original matrix A, if we change a_{34} from 2 to 4, what is the determinant this time?

(e) Suppose we want to change a single entry of A by 1 to make the determinant of the new matrix equal to 2. What entry of A could we change to do this? And to what number should we change it to?

4. For the basis $\mathcal{B} = [(3,5),(1,2)]$ in $\mathbb{R}^2$,

(a) Express $(4,3)$, $(1,2)$, and $(1,3)$ in $\mathcal{B}$-coordinates.

(b) Express $(2,0)_{\mathcal{B}}$, $(1,1)_{\mathcal{B}}$, and $(-1,4)_{\mathcal{B}}$ in the standard coordinates.