DUE DATE: MAR. 17, 2017

1. Put the following complex numbers in the form $a + bi$ (with a, $b \in \mathbb{R}$) :

(a) $(4 - 5i) - (6 - 3i)$ (b) $(2 - 3i) \cdot (4 + 5i)$ (c) $\dfrac{3 - 4i}{6 + 8i}$ (d) $(1 + i)^{20}$

2. Find all the solutions to $z^6 = 1$, where z is a complex number. Draw the solutions on the unit circle in $\mathbb{C}$. Is there one solution whose powers give you all the rest? Is there more than one?

3. You can work out sin and cos of the angles $\pi/4$ and $\pi/3$ (or $\pi/6$) by looking at some special triangles, and most people know these values. But, there are actually more values that can be worked out exactly. In this question, let's work out $\sin(2\pi/5)$ and $\cos(2\pi/5)$.

To make the notation easier, let $c = \cos(2\pi/5)$ and $s = \sin(2\pi/5)$. Let $z = c + s \cdot i$.

(a) Explain why $z^5 = 1$.

(b) Expand $(c + s \cdot i)^5$ and collect the parts with i (the *imaginary part*). Explain why this will be zero, so to find values of c and s, we can start by looking for values where this polynomial in c and s is zero.

In fact, since s isn't zero, and since the above expression is divisible by s, we can divide it by s to get a degree four polynomial in c and s.

(c) Explain why $s^2 = 1 - c^2$, and substitute it into the polynomial above to get a polynomial of degree 4 which only involves the variable c.

(d) If you look more closely, the polynomial from part (c) can also be considered as a polynomial in c^2, of degree 2 (i.e., you can write everything in terms of c^2). Use the quadratic formula to solve for c^2, and then take square roots to find the four possible solutions in c to this polynomial.

(e) To decide which of the four values above is really $\cos(2\pi/5)$, draw a picture of z on the unit circle, and also draw a picture of the point at the angle $\pi/4$. From the picture, explain how you know that $0 < \cos(2\pi/5) < 1/\sqrt{2}$. There is only one solution in (d) which satisfies this condition, so that must be $\cos(2\pi/5)$. Which is it?

(f) Use the formula $s^2 = 1 - c^2$ to find $\sin(2\pi/5)$.

4. Let A be the matrix $\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$.

(a) Compute A^2, A^3, A^4, A^5, and A^6.

(b) What will A^7, A^8, and A^9 be?

(c) If we compute A^k for $k = 1, 2, 3, 4, \ldots$, how many different matrices will we get?

(d) Find the characteristic polynomial of A, and compute its (complex!) eigenvalues.

(e) Looking at the answers to question (2), explain why A has the behaviour above.

[Thinking about the fact that there is a formula for the entries of A^k in terms of the eigenvalues of A, or what A would look like in diagonal form is one way to think about (e), but there are others.]