1. Find and sketch (or describe in words) the domain of the function

$$f(x, y) = \ln(x/y) + \ln(y/x).$$

2. Sketch the following:

(a) The vector field $\mathbf{F}(x, y) = (y, -x)$

(b) The graph of $f(x, y) = e^{-x^2-y^2}$.

3. The voltage at a point $(x, y)$ on a metal plate placed in the $xy$-plane is given by the function $V(x, y) = \sqrt{1 - 9x^2 - 4y^2}$. Sketch the equipotential curves (i.e., the curves of constant voltage).

4. This question concerns limits and continuity.

(a) Using the $\epsilon$–$\delta$ definition of the limit, prove that

$$\lim_{(x,y)\to(5,3)} x = 5 \text{ and } \lim_{(x,y)\to(5,3)} y = 3.$$

(b) Are the functions $\mathbf{F}(x, y) = x$ and $\mathbf{G}(x, y) = y$ continuous at the point $(5, 3)$? Explain why (i.e., justify your answer by showing either that $\mathbf{F}$ and $\mathbf{G}$ satisfy the definition of continuity or showing that they don't).

(c) Compute $\lim_{(x,y)\to(5,3)} 3x^2y - 4y^3 + 2xy - 6$. You do not have to use the $\epsilon$–$\delta$ definition to compute this limit, but rather you should explain how to use your answers from $(a)$ and $(b)$, along with theorems we know, to deduce the limit.

For each of the following questions, compute the limits if they exist. If you claim the limit does exist, briefly (using the properties of limits) justify why. If the limit doesn't exist, explain how you know this.

(d) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\cos(x) - 1 - x^2/2}{x^4 + y^4}$

(e) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{(x - y)^2}{x^2 + y^2}$

5. For each of the following two functions from $\mathbb{R}^2$ to $\mathbb{R}$, find the domain of the function as it is described. If the domain is not all of $\mathbb{R}^2$, is there a way to define the function at the missing points so that the resulting function is continuous everywhere? If so, explain. If not, explain that too.

(a) $f(x, y) = \dfrac{xy}{x^2 + y^2}$.

(b) $g(x, y) = \dfrac{\sin(x + y)}{x + y}$.