Math 280 Homework Assignment 2

DUE DATE: SEPT. 28, 2017

1. Sketch the region $U = \{(x, y) \mid |y| \leqslant \sin(x)\}$, and describe the interior points and the boundary points.

2. Let $u(x, y, t) = e^{-2t}\sin(3x)\cos(2y)$ denote the vertical displacement of a vibrating membrane from the point (x, y) in the xy-plane at time t. Compute $u_x(x, y, t)$, $u_y(x, y, t)$, and $u_t(x, y, t)$ and give physical interpretations of these results.

3. Let $\mathbf{F} : \mathbb{R}^2 \to \mathbb{R}^3$ be the function

$$\mathbf{F}(x, y) = \Big(\sin(\pi x)\cos(\pi y),\ ye^{xy},\ x^2 + y^3\Big).$$

Compute the derivative matrix $\mathbf{DF}$ at $(1, 2)$. If we go through $(1, 2)$ with velocity vector $\vec{v} = (3, -2)$, what are the instantaneous rates of change of the functions $\sin(\pi x)\cos(\pi y)$, ye^{xy}, and $x^2 + y^3$?

4. I'd like to know if the function

$$\mathbf{F}(x, y) = \begin{cases} \dfrac{3xy^2}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\[2ex] 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

is differentiable at $(0, 0)$. We already know that it's continuous at $(0, 0)$ – we did that in class.

(a) If we restrict the function $\mathbf{F}(x, y)$ to the x-axis, points of the form $(x, 0)$, what does the function look like? Is $\mathbf{F}_x(0, 0)$ defined? If so, what does it equal?

(b) Similarly, if we restrict the function $\mathbf{F}(x, y)$ to the y-axis, points of the form $(0, y)$, what does the function look like? Is $\mathbf{F}_y(0, 0)$ defined? If so, what does it equal?

(c) If $\mathbf{F}$ were differentiable at $(0, 0)$, what would its derivative matrix $\mathbf{DF}$ at $(0, 0)$ have to be?

(d) Using that matrix, what would be the instantaneous rate of change of $\mathbf{F}$ at $(0, 0)$ going in the direction $\vec{v} = (1, 1)$?

(e) If we restrict the function to the line $y = x$, points of the form (t, t), what does the function look like? How fast is this function changing when $t = 0$?

(f) If $\mathbf{F}$ were differentiable, explain how the answers to parts (d) and (e) should be related.

(g) Is $\mathbf{F}$ differentiable at $(0, 0)$?

5. Consider the function $\mathbf{F}(x, y) = 25 - x^2 - 2y^2$.

(a) Compute $\mathbf{F}(2,3)$, $\mathbf{F}_x(2,3)$ and $\mathbf{F}_y(2,3)$.

The equation of a plane in $\mathbb{R}^3$ is $z = mx + ny + c$, which we could also consider to be the graph of the function $\mathbf{G}(x, y) = mx + ny + c$.

(b) Compute $\mathbf{G}_x(2,3)$ and $\mathbf{G}_y(2,3)$.

If we want the plane $z = mx + ny + c$ to approximate the graph of $z = \mathbf{F}(x, y)$ above $(2,3)$ as closely as possible, clearly we'd want:

(*i*) The plane to pass through the same point as the graph of $\mathbf{F}(x, y)$ over $(2,3)$.

(*ii*) The plane to have the same instantaneous change in the x-direction as the graph at $(2,3)$.

(*iii*) The plane to have the same instantaneous change in the y-direction as the graph at $(2,3)$.

So,

(c) Find values of m, n, and c so that all these three things are true.

Suppose we pick numbers v_x and v_y and make the vector $\vec{v} = (v_x, v_y)$. The line

$$(2 + t\,v_x,\ 3 + t\,v_y) \qquad \text{for } t \in \mathbb{R}$$

is a line which passes through $(2,3)$ with velocity $\vec{v}$ when $t = 0$.

(d) Compute the function $\mathbf{G}(2 + t\,v_x, 3 + t\,v_y)$ of t, and find its derivative when $t = 0$. (Use the values of m, n, and c from part (c).)

(e) Compute the function $\mathbf{F}(2 + t\,v_x, 3 + t\,v_y)$ of t, and find its derivative when $t = 0$.

(f) Do the answers to (d) and (e) explain what it means for $\mathbf{F}$ to be differentiable at $(2,3)$? Is $\mathbf{F}$ differentiable at $(2,3)$?

(g) (Mini-bonus question) Can you explain why the derivative $\mathbf{DF}$ of a function $\mathbf{F} : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ at a point $(x_1, \dots, x_n)$ should be a linear transformation?