1. Describe the volume being integrated over, and compute the iterated integral.

(a) $\displaystyle\int_0^5 \int_0^{\sqrt{25-x^2}} \int_{\sqrt{x^2+y^2}}^5 z\,dz\,dy\,dx$ (b) $\displaystyle\int_0^{2\pi} \int_0^{1+\sin(x)} \int_0^{1+\sin(x)} 2z\,dz\,dy\,dx$

2. Sketch the region of integration for the iterated integral $\displaystyle\int_0^1 \int_x^1 \int_0^{y-x} f\,dz\,dy\,dx$, and express it in the five other possible orders of integration.

SUGGESTIONS: (1) The region being integrated over is a tetrahedron, and it may help to work out its vertices. (2) Besides a 3D sketch, it will also be helpful to sketch the projection of the region on the xy-, xz-, and yz-planes.

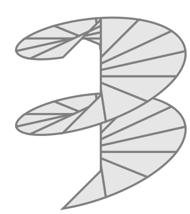
3. For each of the following surfaces, find a parameterization, and compute the tangent vectors and normal vectors in terms of that parameterization:

(a) The graph of $f(x,y) = 9 - xy$ over the circle $x^2 + y^2 \leqslant 9$.

(b) The part of the sphere $x^2 + y^2 + z^2 = 4$ above the plane $z = 1$ (i.e., with $z \geqslant 1$).

(c) The surface obtained by rotating the line segment connecting $(1,0,2)$ to $(4,0,0)$ about the z-axis.

4. Let S be the helicoid parameterized by $(v\cos(\theta), v\sin(\theta), \theta)$ with $(\theta, v) \in [0, 4\pi] \times [0,1]$. A picture of the helicoid is shown at right.



(a) Find the area of S (equivalently, find $\iint_S 1\,dS$).

(b) Find the average value of the function $f(x,y,z) = yz$ over S.

The antiderivative $\int \sqrt{1+u^2}\,du = \frac{1}{2}\left(u\sqrt{1+u^2} + \ln\left(u + \sqrt{1+u^2}\right)\right)$ may be useful in this question.

5. Fix a radius $r > 0$ and two angles φ_1 and φ_2, with $-\frac{\pi}{2} \leqslant \varphi_1 \leqslant \varphi_2 \leqslant \frac{\pi}{2}$. Find the surface area of the portion of the sphere of radius r with lattitudes between φ_1 and φ_2.