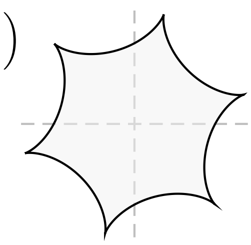
Math 280 — Homework Assignment 10

DUE DATE: NOV. 23, 2017

1. Find the integral $\iint_S \mathbf{F} \cdot dS$ where S is the helicoid parameterized by $(u\cos(v),\ u\sin(v),\ v)$, $0 \leqslant u \leqslant 3$, $0 \leqslant v \leqslant 4\pi$ with positive orientation upwards, and where $\mathbf{F}$ is the vector field $\mathbf{F}(x, y, z) = (xz, -yz, xy)$.

2. Find the flux integral of $\mathbf{F}(x, y, z) = (x^2, y^2, z^2)$ through the top half of the unit sphere, with outward orientation.

3. The parameterized curve $\mathbf{c}(t) = \Big(5\cos(t) + \sin(5t),\ 5\sin(t) + \cos(5t)\Big)$ for $t \in [0, 2\pi]$ is shown at right. Use the vector field $\mathbf{F} = \frac{1}{2}(-y, x)$ and Green's theorem to find the area enclosed by the curve.



The angle addition formula $\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta)$ may prove useful at some point in the calculation.

4. Compute the following line integrals by using Green's Theorem to convert each of them into an integral over a two-dimensional region R, and then evaluating that integral over R.

(a) Compute $\int_{\mathbf{c}} \mathbf{F} \cdot ds$, where $\mathbf{c}$ is the circle of radius 2, centered at $(0, 0)$, oriented counterclockwise, and $\mathbf{F}(x, y) = \Big(\cos(\cos(x)) - x^2y,\ e^{\sin(y^2)} + xy^2\Big)$.

(b) Compute $\int_{\mathbf{c}} \mathbf{F} \cdot ds$, where $\mathbf{c}$ is the boundary of $[1, 2] \times [-1, 1]$, oriented counterclockwise, and $\mathbf{F}(x, y) = \Big(xy^2 + x^3,\ e^{x^2} + e^{y^2}\Big)$.

(c) Compute $\int_{\mathbf{c}} \mathbf{F} \cdot ds$, where $\mathbf{c}$ is the boundary of the region between $y = x^2 - 4x$ and $y = 5$, oriented counterclockwise, and $\mathbf{F}(x, y) = \Big(y, x^2y\Big)$.

5. Consider the following integral, which does not seem very easy to evaluate.

$$(*) \qquad \frac{1}{\pi}\int_0^{2\pi} e^{100\cos^2(t)}\sin(1+e^{30\cos^2(t)})\sin(t)+\cos^2(t)\,dt$$

In this problem we will evaluate the integral by using Green's theorem. Let $\mathbf{c}$ be the circle of radius 1 centered at $(0,0)$, and oriented counterclockwise. One possible parameterization of $\mathbf{c}$ is $\mathbf{c}(t) = (\cos(t), \sin(t))$ with $t \in [0, 2\pi]$.

(a) Find a vector field $\mathbf{F}$ so that when evaluating $\int_{\mathbf{c}} \mathbf{F}\cdot ds$ using the parameterization above, the integral that results is (*).

(b) Use Green's theorem to convert this to an integral over the unit disc, and evaluate that integral.