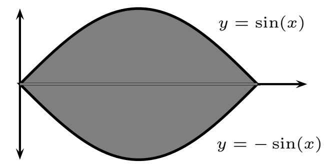
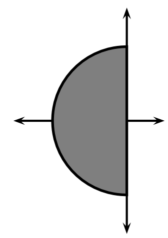
1.

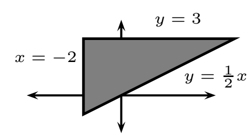
(a) $$\iint_R f(x,y)\,dA = \int_0^{\pi}\int_{-\sin(x)}^{\sin(x)} f(x,y)\,dy\,dx$$



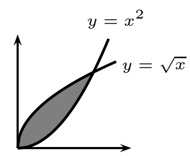
(b) $$\iint_R f(x,y)\,dA = \int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x,y)\,dy\,dx = \int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{0} f(x,y)\,dx\,dy$$



(c) $$\iint_R f(x,y)\,dA = \int_{-2}^{3}\int_{\frac{1}{2}x}^{3} f(x,y)\,dy\,dx = \int_{-2}^{\frac{3}{2}}\int_{-2}^{2y} f(x,y)\,dx\,dy$$



(d) $$\iint_R f(x,y)\,dA = \int_0^1\int_{x^2}^{\sqrt{x}} f(x,y)\,dy\,dx = \int_0^1\int_{y^2}^{\sqrt{y}} f(x,y)\,dy\,dx$$



2.

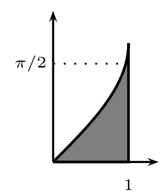
(a)
$$\begin{aligned}
\iint_R e^{-x-3y}\,dA &= \int_0^{\ln 2}\int_0^{\ln 3} e^{-x-3y}\,dy\,dx \\
&= \left(\int_0^{\ln 2} e^{-x}\,dx\right)\left(\int_0^{\ln 3} e^{-3y}\,dy\right) \\
&= \left(1-\frac{1}{2}\right)\frac{1}{3}\left(1-\frac{1}{27}\right) = \frac{13}{81}
\end{aligned}$$

(b)
$$\begin{aligned}
\iint_R xye^{x^2}\,dA &= \int_0^1\int_{-1}^1 xye^{x^2}\,dx\,dy \\
&= \int_0^1 \left(\frac{y}{2}e^{x^2}\right)_{x=-1}^{x=1}\,dy = \int_0^1 0\;\,dy = 0.
\end{aligned}$$

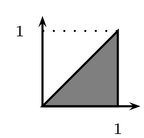
(c) $$\iint_R \ln(xy)\,dA = \int_1^2 \int_{x+1}^3 \ln(xy)\,dy\,dx$$
$$= \int_1^2 \Big(y(\ln(y)-1) + y\ln(x)\Big)_{y=x+1}^{3}\,dx$$
$$= \int_1^2 3\ln(3) - 2 + x + (2-x)\ln(x) - (x+1)\ln(x+1)\,dx$$
$$= \left((3\ln(3)-2)x + \frac{1}{2}x^2 + 2x(\ln(x)-1) - x^2(\frac{\ln(x)}{2} - \frac{1}{4}) - (x+1)^2(\frac{\ln(x+1)}{2} - \frac{1}{4})\right)_{x=1}^{x=2}$$
$$= 3\ln(3) - 2 + \tfrac{1}{2}(4-1) + 4(\ln(2)-1) - 2(0-1) - 4(\tfrac{\ln(2)}{2} - \tfrac{1}{4}) + 1(-\tfrac{1}{4})$$
$$- 9(\tfrac{\ln(3)}{2} - \tfrac{1}{4}) + 4(\tfrac{\ln(2)}{2} - \tfrac{1}{4})$$
$$= 4\ln(2) - \tfrac{3}{2}\ln(3) - \tfrac{1}{2} \cong 0.6246702880\ldots$$

3.

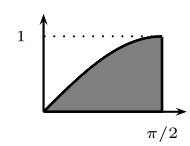
(a) $$\int_0^1 \left(\int_0^{\arcsin(x)} y^2\,dy\right) dx = \int_0^{\pi/2} \int_{\sin(y)}^1 y^2\,dx\,dy = \int_0^{\pi/2} y^2 - y^2\sin(y)\,dy$$
$$= \left(\frac{y^3}{3} - y^2\cos(y) + 2y\sin(y) + 2\cos(y)\right)_{y=0}^{y=\pi/2} = \frac{\pi^3}{24} + \pi - 2.$$



(b) $$\int_0^1 \left(\int_y^1 e^{x^2}\,dx\right) dy = \int_0^1 \int_0^x e^{x^2}\,dy\,dx = \int_0^1 \left(ye^{x^2}\right)_{y=0}^{y=x} dx$$
$$= \int_0^1 xe^{x^2}\,dx = \left(\frac{1}{2}e^{x^2}\right)_{x=0}^{x=1} = \frac{1}{2}(e-1).$$



(c) $$\int_0^1 \left(\int_{\arcsin(y)}^{\pi/2} y\cos(x)\,dx\right) dy = \int_0^1 \int_0^{\sin(x)} y\cos(x)\,dy\,dx = \int_0^1 \left(\frac{y^2}{2}\cos(x)\right)_0^{\sin(x)} dx$$
$$= \frac{1}{2}\int_0^1 \sin^2(x)\cos(x)\,dx = \frac{1}{6}\sin^3(x)\Big|_{x=0}^{x=1} = \frac{\sin^3(1)}{6}.$$



(d) $$\int_{1/2}^{1}\left(\int_{1}^{2y}\frac{\ln x}{x}\,dx\right)\,dy+\int_{1}^{2}\left(\int_{y}^{2}\frac{\ln x}{x}\,dx\right)\,dy=\int_{1}^{2}\int_{\frac{1}{2}x}^{x}\frac{\ln(x)}{x}\;dy\,dx$$

$$=\int_{1}^{2}\left(y\frac{\ln(x)}{x}\right)_{y=\frac{1}{2}x}^{y=x}\;dx=\frac{1}{2}\int_{1}^{2}\ln(x)\,dx=\frac{1}{2}\;x(\ln(x)-1)\Big|_{x=1}^{x=2}$$

$$=\ln(2)-\frac{1}{2}\cong 0.1931471806\ldots$$