DUE DATE: NOV. 24, 2004

1. Compute the integral $\iint_S f\,dS$ for the function $f(x,y,z) = xy$ and the surface S which is the graph of $z = x^2 + y^2$ inside the rectangle $0 \le x \le 3$, $0 \le y \le 2$.

2. Find the surface area of the portion of the sphere of radius r which is described by $\phi_1 \le \phi \le \phi_2$. (Here ϕ is the angle with the z-axis, as in the usual spherical coordinates).

3. Find the integral $\iint_S \mathbf{F}\cdot dS$ where S is the helicoid parameterized by $(u\cos(v),\ u\sin(v),\ v)$, $0 \le u \le 1$, $0 \le v \le 4\pi$ with positive orientation upwards, and $\mathbf{F}$ is the vector field $\mathbf{F}(x,y,z) = (y, -x, xz)$.

4. Find the flux integral of $\mathbf{F}(x,y,z) = (z, x, y^2)$ through the top half of the unit sphere, with outward orientation.

5. Compute the flux integral of

$$\mathbf{F}(x,y,z) = \left(\frac{x}{(x^2+y^2+z^2)^{3/2}}, \frac{y}{(x^2+y^2+z^2)^{3/2}}, \frac{z}{(x^2+y^2+z^2)^{3/2}} \right)$$

through the sphere of radius r, oriented outwards.

Compute the divergence $\mathrm{Div}(\mathbf{F})$ of $\mathbf{F}$. Don't these two answers contradict the divergence theorem? Can you resolve this conflict?