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1. If $\mathbf{c}$ is the top half of the unit circle, oriented from $(-1,0)$ to $(1,0)$, and $\mathbf{F}$ is the vector field

$$\mathbf{F}(x,y) = (\ln(x+5) + y^2, 2xy - y^2),$$

check that $\mathrm{Curl}(\mathbf{F}) = 0$ (the $\mathbb{R}^2$ curl), and use the flexibility theorem for curves to compute the integral $\int_{\mathbf{c}} \mathbf{F} \cdot ds$. Flexing the curve to the line joining $(-1,0)$ and $(1,0)$ is a possibility.

2. If $\mathbf{F}$ is the vector field

$$\mathbf{F}(x,y,z) = (ye^{z^2} - xe^{xy},\ ye^{xy} + \tan(z^2+z+1),\ x^2),$$

check that $\mathrm{Div}(\mathbf{F}) = 0$, and use the flexibility theorem for surfaces to compute $\iint_S \mathbf{F} \cdot dS$ where S is the top half of the unit sphere oriented outwards. ("flexing" it to the unit disk seems like a good bet).

3. If $\mathbf{c}_1$ is the top half of the unit circle, oriented from $(-1,0)$ to $(1,0)$, and $\mathbf{c}_2$ is the line segment joining $(-1,0)$ to $(1,0)$, and $\mathbf{F}$ is the vector field

$$\mathbf{F}(x,y) = (x^2 - y,\ xy - \arcsin(y) + e^{y^3}),$$

use Green's theorem to compute the difference between $\int_{\mathbf{c}_1} \mathbf{F} \cdot ds$ and $\int_{\mathbf{c}_2} \mathbf{F} \cdot ds$.

Use this to compute $\int_{\mathbf{c}_1} \mathbf{F} \cdot ds$ (by computing the easier $\int_{\mathbf{c}_2} \mathbf{F} \cdot ds$, of course…).

4. Could $\mathbf{E}(x,y,z) = (x^2 - y^2,\ e^y - 2z,\ z^2 + 3\sin(y))$ be an electric field (in a static situation)?

Could $\mathbf{B}(x,y,z) = (\sin(x), \sin(y), z^2)$ be a magnetic field (in a static situation)?