DUE DATE: OCT. 27, 2004

1. If $f$ is a function, and $\mathbf{F}$ a vector field on $\mathbb{R}^3$, prove that

$$\mathrm{Div}(f\mathbf{F}) = f\,\mathrm{Div}(\mathbf{F}) + \mathbf{F} \cdot \mathrm{grad}(f).$$

2. Find a parameterization of the curve $x^{2/3} + y^{2/3} = 1$. Is your parameterization continuous? Differentiable? Piecewise $C^1$? Completely $C^1$?

3. Compute the integral of $f(x, y) = xy - x - y + 1$ along the following curves connecting the points $(1, 0)$ and $(0, 1)$.

(a) $\mathbf{c}_1$ : circular arc $\mathbf{c}_1(t) = (\cos(t), \sin(t))$, $0 \le t \le \pi/2$.

(b) $\mathbf{c}_2$ : straight line segment $\mathbf{c}_2(t) = (1 - t, t)$, $0 \le t \le 1$.

(c) $\mathbf{c}_3$ : from $(1, 0)$ horizontally to the origin, then vertically to $(0, 1)$.

(d) $\mathbf{c}_4$ : from $(1, 0)$ vertically to $(1, 1)$, then horizontally to $(0, 1)$.

(e) $\mathbf{c}_5$ : circular arc $\mathbf{c}_5(t) = (\cos(t), -\sin(t))$, $0 \le t \le 3\pi/2$.

4. Find the integral $\int_{\mathbf{c}} f\,ds$ where $f(x, y, z) = \sqrt{9xz + 4y + 1}$ and $\mathbf{c}$ is the "twisted cubic:" $\mathbf{c}(t) = (t, t^2, t^3)$ with $t \in [0, 4]$.

5. Find the average value of the function $f(x, y, z) = xyz$ along the helix

$$\mathbf{c}(t) = (\sin(t), 8t, \cos(t)), \quad t \in [0, 6\pi]$$