DUE DATE: Nov. 17, 2004

1. This problem is related to the change of variables example from class.

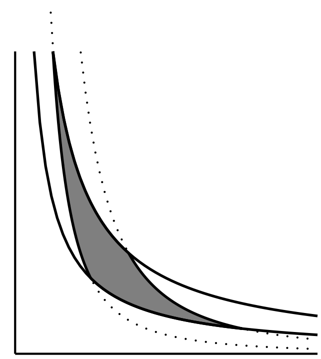
   (a) If for any two positive numbers $a$ and $b$, show either geometrically or algebraically that there is only one solution $(x, y)$ in the positive quadrant to $xy = a$ and $y = bx$.

   (b) The previous part shows that the functions $xy$ and $y/x$ are good coordinates on the positive quadrant – they're sufficient to distinguish any two points in that quadrant. If $xy = u$ and $y/x = v$ are these new coordinates, find formulas for $x$ and $y$ in terms of $u$ and $v$.

   (c) Sketch the region $R$ in the $xy$-plane bounded by $xy = 1$, $xy = 3$, $y = x$ and $y = 4x$. How would you describe $R$ in terms of $u$, $v$ coordinates?

   (d) Compute the Jacobian of the change of coordinates (the determinant of the derivative matrix).

   (e) If $f(x, y)$ is the function $f(x, y) = x^3y^7$, find the integral $\iint_R f(x, y)\, dA$ by changing to $u$, $v$ coordinates.



2. In the picture at right, the solid lines are the curves $xy = 1$ and $xy = 2$, while the dotted lines are the curves $x^2y = 1$ and $x^2y = 3$. Let $R$ be the shaded region between these curves.

The purpose of this question is to compute $\iint_R f(x, y)\, dA$ where $f(x, y) = x^2y^2$.

   (a) Find parameterizations $x(u, v)$ and $y(u, v)$ in terms of $u$ and $v$ so that $x^2y = u$ and $xy = v$.

   (b) In terms of the $u$, $v$ parameterization, what are the limits of integration?

   (c) What is the function $f$ in terms of $u$ and $v$?

   (d) Compute the Jacobian of this parameterization.

   (e) Write down and compute the integral in terms of the $u$ and $v$ parameterization. (i.e., use the change of variables theorem to compute the integral above.)

3. Describe the volume being integrated over, and compute the iterated integral

(a) $\displaystyle\int_{-\sqrt{8}}^{\sqrt{8}} \int_{-\sqrt{8-x^2}}^{\sqrt{8-x^2}} \int_{-3}^{8-x^2-y^2} 2\, dz\, dy\, dx$ (b) $\displaystyle\int_0^1 \int_0^{\sqrt{1-y^2}} \int_0^{\sqrt{1-x^2-y^2}} (2x-y)\, dz\, dx\, dy$

4. Sketch the region of integration for the iterated integral $\displaystyle\int_0^1 \int_0^y \int_0^x x^2yz\, dz\, dx\, dy$, and express it in the five other possible orders of integration.

5. For each of the following surfaces, find a parameterization, and compute the tangent vectors and normal vectors in terms of that parameterization:

(a) The graph of $f(x,y) = 9 - x^2 - y^2$ over the points where the function is positive.

(b) The part of the paraboloid $z = x^2 + y^2$ in the first octant.

(c) The surface obtained by rotating the circle $(y-3)^2 + z^2 = 1$, $x = 0$ about the $z$-axis.