1. Write down the augmented matrices for each of the following systems of equations (writing down the matrices is the entire problem – you don't have to solve the equations).

(a)
$$\begin{array}{rcrcrcrcr} 5x_1 & + & 3x_2 & - & 7x_3 & + & 4x_4 & = & 7 \\ 3x_1 & - & 7x_2 & - & 7x_3 & - & 2x_4 & = & 4 \\ x_1 & + & 2x_2 & + & 2x_3 & - & 5x_4 & = & 2 \end{array}$$

(b)
$$\begin{array}{rcrcrcr} 4a & + & 3b & - & 2c & = & 1 \\ b & - & 2a & - & 6c & = & 4 \\ 3a & + & 7c & + & 5b & = & 0 \end{array}$$

(c)
$$\begin{array}{rcrcrcr} 2x_1 & - & 4x_2 & - & 5x_3 & = & 7 \\ x_1 & & & + & 2x_3 & = & -4 \\ -3x_1 & + & 5x_2 & + & x_3 & = & 2 \\ & - & x_2 & - & 3x_3 & = & 8 \end{array}$$

2. Solve this problem from a 6th century Greek text:

> *Make a crown weighing 60 minae mixing gold, bronze, tin, and iron. Let the gold and bronze together form two thirds, the gold and tin together three fourths, and the gold and iron three fifths. Tell me how much gold, tin, bronze, and iron you must put in.*

Note that a minae is a unit of weight (it doesn't matter what its metric value is, you can just use it as an unknown unit), and that the proportions in the puzzle also refer to weight.

3. Find a polynomial of degree 3 [a polynomial of the form $f(t) = a + bt + ct^2 + dt^3$] whose graph passes through the points $(-1, 10)$, $(0, 2)$, $(1, 0)$, and $(2, 10)$.

4. Put these matrices into row reduced echelon form:

(a) $\begin{bmatrix} 3 & 2 & 1 \\ 6 & 5 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 2 & 1 \\ 7 & 5 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 2 & 1 \\ 6 & 4 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 10 & 6 & 9 \\ 15 & 9 & 12 \end{bmatrix}$