DUE DATE: OCT. 4, 2005

1. List the different types of RREF's possible for a 2×4 matrix. (like we did in class for 2×3 matrices).

BONUS QUESTION: How many types of RREF's are there for a $2 \times n$ matrix?

2. In each of the following four cases, write the vector w as a linear combination of the other vectors.

(a) $w = (1, 2)$; $v_1 = (3, 5)$ and $v_2 = (4, 7)$.

(b) $w = (1, 1)$; $v_1 = (1, 2)$, $v_2 = (3, 4)$.

(c) $w = (3, 6, 11)$; $v_1 = (1, 2, 4)$, $v_2 = (-1, 1, 0)$, and $v_3 = (0, 1, 1)$.

(d) $w = (9, 5, 22)$; $v_1 = (3, 2, 7)$, $v_2 = (2, 1, 5)$, and $v_3 = (0, 1, -1)$.

3. Pigeons are sold at a rate of 5 for 3 coins, swans at a rate of 7 for 5 coins, and peacocks at the rate of 1 for 3 coins. A man was told to bring 100 birds costing 100 coins for the amusement of the King's son. How many of each kind does he buy? (This is a version of a problem from a 9th century Indian text.)

(a) If x is the number of pigeons, y the number of swans, and z the number of peacocks, write down the corresponding linear equations and find the general solutions where x, y, and z are real numbers.

(b) Now solve the problem of the riddle. Since x, y, and z are the numbers of birds, they must be integers, greater than or equal to zero. Also, according to the conditions of the problem, x must be a multiple of 5, and y a multiple of 7. There is more than one solution; find them all.

4. For what values of k and ℓ does the augmented system

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 2 & k & 1 & 2 \\ 3 & 4 & k & \ell \end{array}\right]$$

have

(a) Exactly one solution?

(b) No solutions?

(c) Infinitely many solutions?

Find all the solutions in case (c).

5.

(a) Suppose that we have n homogeneous linear equations in m variables, with $n < m$. Explain why there must be a nonzero solution to the equations.

(b) Suppose that $\vec{v}_1, \ldots, \vec{v}_m$ are vectors in $\mathbb{R}^n$, with $m > n$. Explain why the vectors must be linearly dependent.