1. Compute these matrix multiplications:

(a) $\begin{bmatrix} 2 & -1 \\ 5 & 4 \\ 7 & -4 \end{bmatrix} \begin{bmatrix} 2 & -1 & -2 \\ 3 & 5 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -1 & -2 \\ 3 & 5 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 5 & 4 \\ 7 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} -3 & 1 \\ 6 & 5 \end{bmatrix} \begin{bmatrix} 4 & -3 \\ 1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 1 & 2 \\ -2 & -1 & 3 \\ 7 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 8 & 3 & 8 \\ 2 & 1 & 0 \end{bmatrix}$

2. Suppose we have two linear transformations $T_1 : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ and $T_2 : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ given by these formulas:

$$T_1(x, y, z) = \begin{bmatrix} 7x + 3z \\ 2x + 1y + 8z \end{bmatrix}, \text{ and } T_2(u, v) = \begin{bmatrix} 4u + v \\ 2u + 3v \\ -u + 5v \end{bmatrix}$$

(a) Give the formulas for the composite function $T_3 = T_2 \circ T_1$.

(b) Using these formulas, find the matrix $C$ for $T_3$.

(c) Find the matrix $A$ for $T_1$ and $B$ for $T_2$.

(d) Compute the matrix product $BA$ showing the details of how you computed the entries. (You should, of course, get matrix $C$ as an answer.)

3. Determine if the linear transformations described by the following matrices are invertible. If not, explain why, and if so, find the matrix of the inverse transformation.

(a) $\begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 & 6 \\ 0 & 3 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 7 & 3 \\ 9 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$

(e) $\begin{bmatrix} 3 & 1 & 5 \\ 6 & 3 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ (f) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 5 & 6 & 1 & 0 \\ 7 & 10 & 4 & 1 \end{bmatrix}$

4. Suppose that $A$ is the matrix

$$A = \begin{bmatrix} 5 & 2 & 4 \\ 2 & 3 & 1 \\ 5 & 6 & 3 \end{bmatrix}.$$

(a) Find the inverse of $A$.

(b) Explain why, for any values of $a$, $b$, and $c$, the equations

$$\begin{array}{rcrcrcl} 5x & + & 2y & + & 4z & = & a \\ 2x & + & 3y & + & z & = & b \\ 5x & + & 6y & + & 3z & = & c \end{array}$$

always have a unique solution.

(c) Find this unique solution (in terms of $a$, $b$, and $c$).

5. I don't know if you've seen these before, but there are identities among sin and cos concerning angle addition. That is, if $\alpha$ and $\theta$ are two angles, there is a way to write $\sin(\alpha + \theta)$ in terms of sin's and cos's of $\alpha$ and $\theta$, and the same thing for $\cos(\alpha + \theta)$. There are purely geometric proofs of these identies using triangles, but here's a linear algebra proof:

Let $T_\alpha$ be the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ which is rotation counterclockwise by $\alpha$, and $T_\theta$ the counterclockwise rotation by $\theta$.

(a) Write down the standard matrices for $T_\alpha$ and $T_\theta$, explaining your reasoning for $T_\alpha$.

(b) Explain what the linear transformation $T_\alpha \circ T_\theta$ does to $\mathbb{R}^2$.

(c) Compute the matrix for $T_\alpha \circ T_\theta$ by multiplying the matrices for $T_\alpha$ and $T_\theta$.

(d) On the other hand, from the description in part (b), you can write down directly the matrix for $T_\alpha \circ T_\theta$. What is that matrix?

(e) Since the matrices from parts (c) and (d) are equal (they describe the same linear transformation) what identities among sin and cos must be true?

(f) Using a similar idea, find formulas for $\sin(3\theta)$ and $\cos(3\theta)$ in terms of $\sin(\theta)$ and $\cos(\theta)$.

NOTE: The due date for this assignment is now **Thursday, Oct. 20**, since we didn't cover computing the inverse in class last week.