MATH 110 Tutorial 12

Things you should know from before the midterm:

- Dot products, orthogonality, norms, projections
- RREFing, rank
- Linear transformations, injectivity, surjectivity, image, kernel, composition
- Inverses
- Linear combinations, linear independence, spans, bases,
- Subspaces

Things you should know since the midterm:

- The Rank-Nullity Theorem
- The Kernel Principal
- Prime Numbers
- Infinitude of Primes, Unique Factorization
- Wilson's Theorem
- Modular arithmetic
- Fields and inverses modulo p
- Linear algebra over \mathbb{F}_p
- Error Correcting Codes

Practice Problems.

1. Prove the infinitude of primes. One approach is to suppose that there are only finitely many and try to arise at a contradiction.

2. Find a cubic polynomial over \mathbb{F}_{11} that passes through the points (1, 1), (4, 2), (8, 6).

3. Find the kernel of the given matrix A. Find all solutions to Ax = (1, 2, 3).

$$A = \begin{bmatrix} 1 & 3 & 5 & 4 & 0 \\ 1 & -2 & 9 & 2 & 1 \\ 1 & 3 & 1 & 1 & 2 \end{bmatrix}$$

4. The vector (1, 4, 2, 0, 1, 0, 4, x, 5, 1) is an ISBN code for what value of x?

5. Compute 99! mod 101. For goodness sakes, use Wilson's Theorem to help you.

6. Challenge. Prove that for every prime p > 2 there is an element of \mathbb{F}_p besides 1 that is its own inverse. (e.g. $2^{-1} = 2 \mod 3$)