MATH 110 Tutorial 12

Things you should know from before the midterm:

- Dot products, orthogonality, norms, projections
- RREFing, rank
- Linear transformations, injectivity, surjectivity, image, kernel, composition
- Inverses
- Linear combinations, linear independence, spans, bases,
- Subspaces

Things you should know since the midterm:

- The Rank-Nullity Theorem
- The Kernel Principal
- Prime Numbers
- Infinitude of Primes, Unique Factorization
- Wilson's Theorem
- Modular arithmetic
- Fields and inverses modulo p
- Linear algebra over $\mathbb{F}_p$
- Error Correcting Codes

Practice Problems.

1. Prove the infinitude of primes. One approach is to suppose that there are only finitely many and try to arise at a contradiction.

2. Find a cubic polynomial over $\mathbb{F}_{11}$ that passes through the points $(1,1),(4,2),(8,6)$.

3. Find the kernel of the given matrix A. Find all solutions to $Ax=(1,2,3)$.

$$A = \begin{bmatrix} 1 & 3 & 5 & 4 & 0 \\ 1 & -2 & 9 & 2 & 1 \\ 1 & 3 & 1 & 1 & 2 \end{bmatrix}$$

4. The vector $(1,4,2,0,1,0,4,x,5,1)$ is an ISBN code for what value of x ?

5. Compute $99!$ mod 101. For goodness sakes, use Wilson's Theorem to help you.

6. *Challenge.* Prove that for every prime $p>2$ there is an element of $\mathbb{F}_p$ besides 1 that is its own inverse. (*e.g.* $2^{-1}=2 \bmod 3$)