MATH 110 Tutorial 3

Definition. A matrix is in *row echelon form* if it satisfies

1. Any rows consisting of zeros are at the bottom, and
2. In each nonzero row, the first nonzero entry (called a *leading entry*) is in a column to the left of any leading entries below it.

Definition. A matrix is in *reduced row echelon form* if it satisfies

1. It is in row echelon form, and
2. The first nonzero entry in each row is 1, and
3. Each column containing a leading 1 has zeros in every other entry.

Proposition. A system of linear equations with real coefficients has either:

- a unique solution,
- infinitely many solutions, or
- no solutions (inconsistent system).

Definition. The *rank* of a matrix is the number of nonzero rows in its row echelon form. Equivalently, this is the number of leading entries in its row echelon form.

Definition. The matrix A is said to be *row equivalent* to the matrix B if they have the same reduced row echelon form.

1. Reduce the following matrices into reduced row echelon form.

$$(a)\begin{bmatrix} 1 & 0 & 8 \\ 2 & 3 & 3 \\ 4 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix} \quad (b)\begin{bmatrix} -2 & 4 & 2 \\ 3 & -1 & 1 \end{bmatrix} \quad (c)\begin{bmatrix} 1 & 2 \\ 4 & 0 \\ -2 & 1 \\ -1 & 1 \end{bmatrix} \quad (d)\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

2. For each of the above matrices, now consider them as augmented matrices, and write explicitly the solution set for the system of equations they represent.

3. What is the rank of each matrix above? Does the rank help in deciding how many solutions there are to the corresponding system of equations?

4. Solve the system of equations:

$$\begin{aligned} 4x + 2y - z &= 2 \\ -x + 3y - 2z &= 0 \\ 2x - 4y + 5z &= -1 \end{aligned}$$

5. Prove that if a matrix has more columns than rows it has a free variable.

6. *Challenge.* Let A be a 2×2 matrix, and consider the system of equations represented by $A|0$. Determine a simple test for deciding when there is only the solution $x = y = 0$ to the implied system of equations. What about a test for 3×3 ?