MATH 110 Tutorial 6

We compute the inverse of A by augmenting the identity to A and reducing into RREF. Then the augment is the inverse, if it exists.

Suppose T is a linear transformation which can be represented by the matrix A. Then A^{-1}, the inverse matrix of A, represents the inverse linear transformation T^{-1}.

For a linear transformation T represented by the $n \times n$ matrix A, the following are equivalent.

- A is invertible.
- $A\vec{x} = \vec{b}$ has a unique solution $\vec{x} \in \mathbb{R}^n$ for each $\vec{b} \in \mathbb{R}^n$.
- $\text{rref}(A) = I_n$.
- $\text{rank}(A) = n$.
- $\text{image}(T) = \mathbb{R}^n$.
- $\text{kernel}(T) = \{\vec{0}\}$.

The inverse of a product of invertible square matrices is the *reversed* product of the inverses of the factors: $(AB)^{-1} = B^{-1}A^{-1}$.

Practice Problems.

1. Invert the following matrices if possible; for the last matrix, for what k is it invertible?

$$(a) \begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}, \qquad (b) \begin{bmatrix} 1 & -1 & 1 \\ 3 & -2 & 2 \\ -1 & 0 & 3 \end{bmatrix}, \qquad (c) \begin{bmatrix} 1 & 0 & 2 \\ 0 & 4 & -1 \\ 2 & -4 & k \end{bmatrix}$$

2. Let A, B be matrices so that the product AB exists. If $\ker(A) = \ker(B) = \{0\}$, then find $\ker(AB)$.

3. Give a simple example to show that it is *not the case* that the sum of two invertible matrices must be invertible.

4. Suppose that A is invertible. What is the inverse of A^3 ? The matrix B is called **nilpotent** if some power of B is the zero matrix. Prove that a nilpotent matrix cannot be invertible.

5. *Challenge.* A set is said to be **closed under addition** if the sum of two elements of the set is also in the set. An example of a set closed under addition is the integers. Prove that for any linear transformation T, the kernel and image of T are closed under addition.