1. Compute the following determinants

(a) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & 3 \\ 2 & 2 & 5 \end{vmatrix}$ (b) $\begin{vmatrix} 2 & 5 & 9 & 7 \\ 3 & 2 & 1 & 1 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 3 & 5 \end{vmatrix}$ (c) $\begin{vmatrix} 1 & 3 & 2 & 4 \\ 1 & 6 & 4 & 8 \\ 1 & 3 & 0 & 0 \\ 2 & 6 & 4 & 12 \end{vmatrix}$

(d) $\begin{vmatrix} 5 & 2 & 7 & 3 \\ 6 & 2 & 6 & 4 \\ 7 & 4 & 6 & 5 \\ 1 & 6 & 4 & 7 \end{vmatrix}$ (e) $\begin{vmatrix} 3 & 2 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 0 \\ 5 & 2 & 1 & 7 & 1 \\ 0 & 6 & 0 & 0 & 2 \\ 2 & 6 & 3 & 0 & 1 \end{vmatrix}$ (f) $\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 1 & 3 & 3 & 3 \\ 1 & 1 & 1 & 4 & 4 \\ 1 & 1 & 2 & 1 & 5 \end{vmatrix}$

2. Suppose that a, b, c, d, e, and f are numbers such that

$$\begin{vmatrix} a & 1 & d \\ b & 1 & e \\ c & 1 & f \end{vmatrix} = 7 \text{ and } \begin{vmatrix} a & 1 & d \\ b & 2 & e \\ c & 3 & f \end{vmatrix} = 11.$$

Find

(a) $\begin{vmatrix} a & 3 & d \\ b & 3 & e \\ c & 3 & f \end{vmatrix}$ (b) $\begin{vmatrix} a & 3 & d \\ b & 5 & e \\ c & 7 & f \end{vmatrix}$ (c) $\begin{vmatrix} e & b & 5 \\ f & c & 9 \\ d & a & 1 \end{vmatrix}$

3. Just so that we don't forget how to work with finite fields, compute the determinants in (a) and (b) working in $\mathbb{F}_5$, and the determinants in (c) and (d) working in $\mathbb{F}_7$. (The formula for the determinant is the same – you just do all the arithmetic in $\mathbb{F}_5$ and $\mathbb{F}_7$.)

(a) $\begin{vmatrix} 2 & 4 & 1 \\ 1 & 1 & 3 \\ 0 & 1 & 4 \end{vmatrix}$ (b) $\begin{vmatrix} 4 & 2 & 1 & 3 \\ 0 & 3 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 3 & 3 & 1 & 2 \end{vmatrix}$

(c) $\begin{vmatrix} 1 & 2 & 5 \\ 1 & 3 & 1 \\ 6 & 3 & 1 \end{vmatrix}$ (d) $\begin{vmatrix} 6 & 5 & 3 & 2 \\ 3 & 5 & 1 & 4 \\ 6 & 1 & 0 & 2 \\ 5 & 0 & 1 & 1 \end{vmatrix}$

4. Going back to $\mathbb{R}$, let A be the matrix

$$A = \begin{bmatrix} 2 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \\ 1 & 1 & 2 & 2 \end{bmatrix}.$$

(a) By changing only a single entry of A to another integer, make the determinant of the new matrix equal to 2.

How did you arrive at your choice?

(b) Is it possible to change only a single entry of A (again to another integer) so that the determinant of the new matrix is 4? Why or Why not?