DUE DATE: MAR. 27, 2008

NAME THAT RING!

Each of the quotient rings R/I in the leftmost list is isomorphic to one of the rings S in the rightmost list. Match each ring with its isomorphic partner, and prove that they really are isomorphic by describing a surjective ring homomorphism $\phi\colon R \longrightarrow S$ with kernel I. (NOTE: The matching of the left list to the right list is neither injective nor surjective.)

R/I	S
(a) $\dfrac{\mathbb{Z}[x]}{\langle 8, 12, x\rangle}$.	1. $\mathbb{Z}/3\mathbb{Z}$
(b) $\dfrac{\mathbb{Q}[x]}{\langle x^2-2\rangle}$.	2. $\mathbb{Z}/4\mathbb{Z}$
(c) $\dfrac{\mathbb{R}[x]}{\langle x-\sqrt{2}\rangle}$.	3. $\mathbb{Z}/8\mathbb{Z}$
(d) $\dfrac{\mathbb{R}[x]}{\langle x^2+x+2\rangle}$.	4. $\mathbb{Z}$
(e) $\dfrac{\mathbb{R}[x]}{\langle x^2\rangle}$.	5. $\mathbb{Q}$
(f) $\dfrac{\mathbb{R}[x,y]}{\langle y-1, x+9\rangle}$.	6. $\mathbb{R}$
(g) $\dfrac{\mathbb{R}[x,y]}{\langle y-1, x^2+9\rangle}$.	7. $\mathbb{C}$
(h) $\dfrac{\mathbb{R}[x,y]}{\langle xy-1\rangle}$.	8. $\left\{a+b\sqrt{2} \mid a,b\in\mathbb{Q}\right\}$
	9. $\mathbb{Z}[x]$
	10. $\mathbb{Q}[x]$
	11. $\mathbb{R}[x]$
	12. $\mathbb{C}[x]$
	13. $\mathbb{R}[t, \frac{1}{t}]$ (polynomials in t and $\frac{1}{t}$)
	14. $\left\{ \begin{pmatrix} a & b \\ 0 & a \end{pmatrix} \mid a,b\in\mathbb{R}\right\}$

In example 14 the ring operations are addition and multiplication of matrices.

Some methods you might use to try and match up R/I and S: (*i*) guess, (*ii*) think about representatives in the quotient ring, and what the multiplication rules are and try and match that up with something in the S column, (*iii*) try and make up a homomorphism from R to somewhere that would have elements of the ideal I in the kernel.