Math 326 — Homework Assignment 11

DUE DATE: DEC. 2, 2010

1. Let R and ω be positive real numbers, and let γ_R be the half circle $\left\{ z \in \mathbb{C} \,\middle|\, |z| = R,\ \mathrm{Im}(z) \geq 0 \right\}$.

(a) Find the maximum value of $|e^{i\omega z}|$ on γ_R.

(b) Find the maximum value of $|e^{-i\omega z}|$ on γ_R.

(c) Find $\displaystyle\int_{-\infty}^{\infty} \frac{e^{-2xi}}{1+x^4}\, dx$.

2. Compute the following integrals, where ω is a nonzero real number. (You might have to distinguish between the cases $\omega > 0$ and $\omega < 0$).

(a) P.V. $\displaystyle\int_{-\infty}^{\infty} \frac{e^{i\omega x}}{(x-1)(x-2)}\, dx$.

(b) $\displaystyle\int_{-\infty}^{\infty} \frac{e^{i\omega x}}{(x^2+1)^2}\, dx$.

(c) P.V. $\displaystyle\int_{-\infty}^{\infty} \frac{e^{i\omega x}}{x(x^2+1)^2}\, dx$.

3. For a real number $a \neq 0$, compute

(a) P.V. $\displaystyle\int_{-\infty}^{\infty} \frac{\sin(x)}{x-a}\, dx$.

(b) $\displaystyle\int_{-\infty}^{\infty} \frac{\cos(x)}{(x^2+a^2)^2}\, dx$.

4. Let a and b be positive real numbers with $a > b$. Compute

(a) $\displaystyle\int_{0}^{2\pi} \frac{1}{a+b\cos(\theta)}\, d\theta$.

(b) $\displaystyle\int_{0}^{2\pi} \frac{1}{(a+b\cos(\theta))^2}\, d\theta$.

5. One of the conditions of Proposition 30.1 was that the poles on the real axis should all be simple. From this you might guess one of two things might be true: (*i*) there is a more general result dealing with poles of higher order on the real axis, but the corresponding formula is more complicated, or (*ii*) if the poles have higher order on the real axis the principal value doesn't exist (i.e., the limit won't converge).

Fix $z_0 \in \mathbb{C}$, and $\theta_1, \theta_2 \in \mathbb{R}$ and for any $\epsilon > 0$ let γ_ϵ be the arc of radius ϵ around z_0 going from angle θ_1 to angle θ_2.

(a) Compute (as in the proof of Lemma 30.2) $\lim_{\epsilon \to 0} \int_{\gamma_\epsilon} \frac{1}{(z - z_0)^2}\, dz$.

(b) Decide between the two possibilities above. If you decide that (*ii*) is true, explain why the integral must diverge (this might require recalling part of the proof of Proposition 30.1). If you decide that (*i*) is true, give the more general formula for dealing with poles of order 2.