1.

(a) Finding when one vector is a linear combination of some others is a problem we know how to solve: we simply re-express it as the problem of solving a system of linear equations, which we can deal with in the usual way. For instance, to find out how $\vec{b}_1 = (1,0)$ is a linear combination of $\vec{v}_1 = (4,3)$ and $\vec{v}_2 = (1,2)$ is the same as solving the system of linear equations

$$\begin{aligned} 4x + y &= 1 \\ 3x + 2y &= 0. \end{aligned}$$

Which we then put into matrix form and row reduce:

$$\left[\begin{array}{cc:c} 4 & 1 & 1 \\ 3 & 2 & 0 \end{array}\right] \rightsquigarrow \left[\begin{array}{cc:c} 1 & 0 & \frac{2}{5} \\ 0 & 1 & -\frac{3}{5} \end{array}\right]$$

showing us that $\vec{b}_1 = \frac{1}{5}\,\vec{v}_1 - \frac{3}{5}\,\vec{v}_2$.

Similarly $\vec{b}_2 = -\frac{1}{5}\,\vec{v}_1 + \frac{4}{5}\,\vec{v}_2$, and $\vec{b}_3 = 4\,\vec{v}_1 - 5\,\vec{v}_2$. Rather than do three separate row reductions, it's more efficient to do them all at once:

$$\left[\begin{array}{cc:ccc} 4 & 1 & 1 & 0 & 11 \\ 3 & 2 & 0 & 1 & 2 \end{array}\right] \rightsquigarrow \left[\begin{array}{cc:ccc} 1 & 0 & \frac{2}{5} & -\frac{1}{5} & 4 \\ 0 & 1 & -\frac{3}{5} & \frac{4}{5} & -5 \end{array}\right]$$

and then read off the solutions.

(b) The key property we know about T is that it is a linear transformation. That means, for any two vectors $\vec{w}_1$ and $\vec{w}_2$ in $\mathbb{R}^2$,

$$T(\vec{w}_1 + \vec{w}_2) = T(\vec{w}_1) + T(\vec{w}_2),$$

and for any vector $\vec{w}$ in $\mathbb{R}^2$ and any number c,

$$T(c\,\vec{w}) = c\,T(\vec{w})$$

The way we use this is that if we know what T does to a collection of vectors, we then know what T does to any linear combination of those vectors.

For instance, since we know that

$$T\left(\begin{bmatrix} 4 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}, \text{ and } T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix},$$

linearity allows us to calculate that:

$$\begin{aligned} T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) &= T\left(\frac{2}{5}\begin{bmatrix} 4 \\ 3 \end{bmatrix} - \frac{3}{5}\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) \\ &= \frac{2}{5}T\left(\begin{bmatrix} 4 \\ 3 \end{bmatrix}\right) - \frac{3}{5}T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) \\ &= \frac{2}{5}\begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} - \frac{3}{5}\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} -1 \\ 1 \\ \frac{4}{5} \end{bmatrix}. \end{aligned}$$

Similarly we compute that

$$\begin{aligned} T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) &= T\left(-\frac{1}{5}\begin{bmatrix} 4 \\ 3 \end{bmatrix} + \frac{4}{5}\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) \\ &= -\frac{1}{5}T\left(\begin{bmatrix} 4 \\ 3 \end{bmatrix}\right) + \frac{4}{5}T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) \\ &= -\frac{1}{5}\begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} + \frac{4}{5}\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} 2 \\ -1 \\ \frac{3}{5} \end{bmatrix} \end{aligned}$$

and that

$$\begin{aligned}
T\left(\begin{bmatrix} 11 \\ 2 \end{bmatrix}\right) &= T\left(4\begin{bmatrix} 4 \\ 3 \end{bmatrix} - 5\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) \\
&= 4\,T\left(\begin{bmatrix} 4 \\ 3 \end{bmatrix}\right) - 5\,T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) \\
&= 4\begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} - 5\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \\
&= \begin{bmatrix} -7 \\ 9 \\ 10 \end{bmatrix}.
\end{aligned}$$

(c) To find the matrix of a linear transformation starting in $\mathbb{R}^2$, we just have to know where the special vectors $\vec{e}_1$ and $\vec{e}_2$ go, which is just what we've computed above. So, the matrix A for the linear transformation T is

$$A = \begin{bmatrix} -1 & 2 \\ 1 & -1 \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}.$$

(d) Multiplying, we get

$$\begin{bmatrix} -1 & 2 \\ 1 & -1 \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1\begin{bmatrix} -1 \\ 1 \\ \frac{4}{5} \end{bmatrix} + 0\begin{bmatrix} 2 \\ -1 \\ \frac{3}{5} \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ \frac{4}{5} \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 \\ 1 & -1 \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0\begin{bmatrix} -1 \\ 1 \\ \frac{4}{5} \end{bmatrix} + 1\begin{bmatrix} 2 \\ -1 \\ \frac{3}{5} \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ \frac{3}{5} \end{bmatrix}$$

and that

$$\begin{bmatrix} -1 & 2 \\ 1 & -1 \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}\begin{bmatrix} 11 \\ 2 \end{bmatrix} = 11\begin{bmatrix} -1 \\ 1 \\ \frac{4}{5} \end{bmatrix} + 2\begin{bmatrix} 2 \\ -1 \\ \frac{3}{5} \end{bmatrix} = \begin{bmatrix} -7 \\ 9 \\ 10 \end{bmatrix}.$$

2. If we fix the vector $\vec{v} = (v_1, v_2, v_3)$, then for any vector $\vec{x} = (x_1, x_2, x_3)$ the formula for the cross product tells us that

$$\vec{v} \times \vec{x} = \begin{bmatrix} v_2x_3 - v_3x_2 \\ v_3x_1 - v_1x_3 \\ v_1x_2 - v_2x_1 \end{bmatrix}.$$

For instance, if $\vec{x} = (1, 0, 0)$ we get

$$\vec{v} \times \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ v_3 \\ -v_2 \end{bmatrix},$$

or if $\vec{x} = (1, 2, 3)$, we get

$$\vec{v} \times \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3v_2 - 2v_3 \\ v_3 - 3v_1 \\ 2v_1 - v_2 \end{bmatrix}.$$

This gives us a rule for taking a vector in $\mathbb{R}^3$ and producing another vector in $\mathbb{R}^3$, in other words, a function $T_{\vec{v}} : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$. (The subscript $\vec{v}$ is just a reminder that this function depends on our original choice of the vector $\vec{v}$.)

The problem is to show that $T_{\vec{v}}$ is a linear transformation, i.e., that it obeys the two key rules $T_{\vec{v}}(\vec{x}+\vec{y}) = T_{\vec{v}}(\vec{x})+T_{\vec{v}}(\vec{y})$ for any vectors $\vec{x}$ and $\vec{y}$ in $\mathbb{R}^3$, and that $T_{\vec{v}}(c\,\vec{x}) = c\,T_{\vec{v}}(\vec{x})$ for any vector $\vec{x}$ in $\mathbb{R}^3$ and any number c.

One way to do this is to take arbitrary vectors $\vec{x} = (x_1, x_2, x_3)$ and $\vec{y} = (y_1, y_2, y_3)$, with sum $\vec{x} + \vec{y} = (x_1 + y_1, x_2 + y_2, x_3 + y_3)$ and compute what $T_{\vec{v}}$ does to each of them:

$$T_{\vec{v}}(\vec{x}) = \begin{bmatrix} v_2x_3 - v_3x_2 \\ v_3x_1 - v_1x_3 \\ v_1x_2 - v_2x_1 \end{bmatrix}, \qquad T_{\vec{v}}(\vec{y}) = \begin{bmatrix} v_2y_3 - v_3y_2 \\ v_3y_1 - v_1y_3 \\ v_1y_2 - v_2y_1 \end{bmatrix},$$

and

$$T_{\vec{v}}(\vec{x} + \vec{y}) = \begin{bmatrix} v_2(x_2 + y_3) - v_3(x_2 + y_2) \\ v_3(x_1 + y_1) - v_1(x_3 + y_3) \\ v_1(x_2 + y_2) - v_2(x_1 + y_1) \end{bmatrix}.$$

Then to check that $T_{\vec{v}}(\vec{x} + \vec{y}) = T_{\vec{v}}(\vec{x}) + T_{\vec{v}}(\vec{y})$ we just have to see if

$$\begin{bmatrix} v_2(x_2 + y_3) - v_3(x_2 + y_2) \\ v_3(x_1 + y_1) - v_1(x_3 + y_3) \\ v_1(x_2 + y_2) - v_2(x_1 + y_1) \end{bmatrix} \text{ is equal to } \begin{bmatrix} v_2x_3 - v_3x_2 \\ v_3x_1 - v_1x_3 \\ v_1x_2 - v_2x_1 \end{bmatrix} + \begin{bmatrix} v_2y_3 - v_3y_2 \\ v_3y_1 - v_1y_3 \\ v_1y_2 - v_2y_1 \end{bmatrix},$$

which is true.

Similarly, for an arbitrary number c, we could compute what $T_{\vec{v}}$ does to the vector $c\,\vec{x} = (c\,x_1, c\,x_2, c\,x_3)$,

$$T_{\vec{v}}(c\,\vec{x}) = \begin{bmatrix} c\,v_2x_3 - c\,v_3x_2 \\ c\,v_3x_1 - c\,v_1x_3 \\ c\,v_1x_2 - c\,v_2x_1 \end{bmatrix}.$$

Then to check that $T_{\vec{v}}(c\,\vec{x}) = c\,T_{\vec{v}}(\vec{x})$ we just need to see if

$$\begin{bmatrix} c\,v_2x_3 - c\,v_3x_2 \\ c\,v_3x_1 - c\,v_1x_3 \\ c\,v_1x_2 - c\,v_2x_1 \end{bmatrix} \text{ is equal to } c \begin{bmatrix} v_2x_3 - v_3x_2 \\ v_3x_1 - v_1x_3 \\ v_1x_2 - v_2x_1 \end{bmatrix},$$

which is also true.

This shows that $T_{\vec{v}}$ is a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$.

An alternate method of argument, if you've seen the cross product before, is to use the fact that the cross product satisfies the identities:

$$\vec{v} \times (\vec{x} + \vec{y}) = (\vec{v} \times \vec{x}) + (\vec{v} \times \vec{y}), \quad \text{and} \quad \vec{v} \times (c\,\vec{x}) = c\,(\vec{v} \times \vec{x}).$$

On the other hand, the way to establish those identities is to do the the calculations we did above, so it's really the same solution.

3. To find the matrix for any of these linear transformations, we just have to see what happens to the key vectors $\vec{e}_1 = (1,0,0)$, $\vec{e}_2 = (0,1,0)$, and $\vec{e}_3 = (0,0,1)$. Each of the rotations we're interested in has one of the axes as its axis of rotation, so in each of the cases one of the $\vec{e}_i$'s stays where it is, and the other two rotate in the appropriate (xy, yz, or xz) plane. When we try to work out what happens in that plane, it's really just a two dimensional rotation problem again, which we know how to solve.

(a) For instance, in rotation around the z-axis, the vector $\vec{e}_3$ is fixed, and the rotation is really just a rotation in the xy-plane. We know what happens to vectors in the xy-plane if we rotate by $\pi/4$ clockwise. The resulting matrix is:

$$A_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) Rotating around the x axis fixes $\vec{e}_1$ and rotates in the yz-plane. In the plane, the transformation "rotation counterclockwise by $\pi/2$" is easy to figure out – it sends the y-axis to the z-axis, and the z-axis to the negative y-axis:

$$A_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

(c) Finally, rotation around the y axis fixes $\vec{e}_2$, and then we're again just rotating by $\pi/4$ in the xz-plane.

$$A_3 = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

a little care is required to figure out what "clockwise" means when looking down from the positive y-axis at the xz-plane.

(d) The results of applying the transformations in the order T_1, T_2, and T_3 are

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{T_1} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} \xrightarrow{T_2} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix} \xrightarrow{T_3} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \xrightarrow{T_1} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} \xrightarrow{T_2} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} \xrightarrow{T_3} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \xrightarrow{T_1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \xrightarrow{T_2} \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \xrightarrow{T_3} \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

In terms of what we've learned recently, the above calculations are computing the matrix for the composite map $T_3 \circ T_2 \circ T_1$, which is the same thing as the matrix product $A_3A_2A_1$:

$$A_3A_2A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

Can you see geometrically why the composition turned out to be so simple?

4.

(a) It's not immediately obvious what the projection T_m does to the standard vectors $\vec{e}_1$ and $\vec{e}_2$, which is what we need to know to write down the matrix for T_m.

But, there are two vectors in $\mathbb{R}^2$ where it is pretty clear what the projection has to do: for vectors on the line of slope m the projection T_m does nothing, and for vectors perpendicular to the line, the projection T_m sends them to $\vec{0} = (0, 0)$.

A vector which lies on the line of slope m is $\vec{v}_1 = (1, m)$, and a vector perpendicular to this line is $\vec{v}_2 = (m, -1)$. Writing out the previous observations in mathematical notation, we know that

$$T_m\left(\begin{bmatrix} 1 \\ m \end{bmatrix}\right) = \begin{bmatrix} 1 \\ m \end{bmatrix} \quad \text{and} \quad T_m\left(\begin{bmatrix} m \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

In order to see what T_m does to $\vec{e}_1$ and $\vec{e}_2$ we can use the same strategy as in question (1): write $\vec{e}_1$ and $\vec{e}_2$ as linear combinations of $\vec{v}_1$ and $\vec{v}_2$, and use the linearity of T_m to figure out what happens to $\vec{e}_1$ and $\vec{e}_2$.

We can write $\vec{e}_1$ and $\vec{e}_2$ as

$$\begin{aligned}
\begin{bmatrix} 1 \\ 0 \end{bmatrix} &= \frac{1}{1+m^2}\begin{bmatrix} 1 \\ m \end{bmatrix} + \frac{m}{1+m^2}\begin{bmatrix} m \\ -1 \end{bmatrix} \\
\begin{bmatrix} 0 \\ 1 \end{bmatrix} &= \frac{m}{1+m^2}\begin{bmatrix} 1 \\ m \end{bmatrix} - \frac{1}{1+m^2}\begin{bmatrix} m \\ -1 \end{bmatrix}
\end{aligned}$$

which is something we can work out either by inspection, or in the usual way, by row reducing the matrix describing this linear algebra problem

$$\left[\begin{array}{cc:cc} 1 & m & 1 & 0 \\ m & -1 & 0 & 1 \end{array}\right] \rightsquigarrow \left[\begin{array}{cc:cc} 1 & 0 & \frac{1}{1+m^2} & \frac{m}{1+m^2} \\ 0 & 1 & \frac{m}{1+m^2} & -\frac{1}{1+m^2} \end{array}\right].$$

Using the linearity of T, we can now conclude that

$$\begin{aligned}
T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) &= \frac{1}{1+m^2}T\left(\begin{bmatrix} 1 \\ m \end{bmatrix}\right) + \frac{m}{1+m^2}T\left(\begin{bmatrix} m \\ -1 \end{bmatrix}\right) \\
&= \frac{1}{1+m^2}\begin{bmatrix} 1 \\ m \end{bmatrix} + \frac{m}{1+m^2}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{1+m^2} \\ \frac{m}{1+m^2} \end{bmatrix}
\end{aligned}$$

and

$$\begin{aligned}
T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) &= \frac{m}{1+m^2}T\left(\begin{bmatrix} 1 \\ m \end{bmatrix}\right) - \frac{1}{1+m^2}T\left(\begin{bmatrix} m \\ -1 \end{bmatrix}\right) \\
&= \frac{m}{1+m^2}\begin{bmatrix} 1 \\ m \end{bmatrix} - \frac{1}{1+m^2}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{m}{1+m^2} \\ \frac{m^2}{1+m^2} \end{bmatrix}
\end{aligned}$$

and therefore that the matrix A_m for the linear transformation T_m is

$$A_m = \begin{bmatrix} \frac{1}{1+m^2} & \frac{m}{1+m^2} \\ \frac{m}{1+m^2} & \frac{m^2}{1+m^2} \end{bmatrix}.$$

(b) Row reducing:

$$\begin{bmatrix} \frac{1}{1+m^2} & \frac{m}{1+m^2} \\ \frac{m}{1+m^2} & \frac{m^2}{1+m^2} \end{bmatrix} \overset{R2-m\cdot R1}{\rightsquigarrow} \begin{bmatrix} \frac{1}{1+m^2} & \frac{m}{1+m^2} \\ 0 & 0 \end{bmatrix}$$

we see that A_m has rank one. Soon we'll see that this is exactly because the image of T_m is a line, i.e., a one dimensional vector space (the "one"'s are the same one).

(c) We're defining $T'_m(\vec{v})$ by $T'_m(\vec{v}) = \vec{v} - T_m(\vec{v})$. To show that T'_m is linear we have to show that T'_m satisfies the two special conditions: $T'_m(\vec{v}+\vec{w}) = T'_m(\vec{v}) + T'_m(\vec{w})$ for any two vectors $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^2$, and that $T'_m(c\,\vec{v}) = c\,T'_m(\vec{v})$ for any vector $\vec{v}$ in $\mathbb{R}^2$ and any number c.

We know that T_m is linear. Using this fact, and the definition of T'_m, showing that T'_m is also a linear transformation is mostly a formal matter:

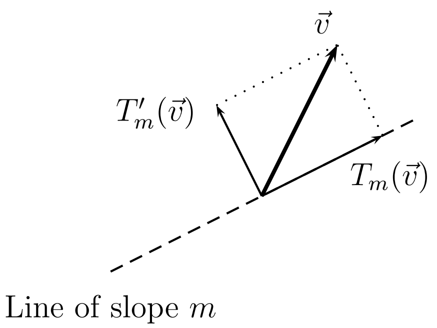
$$\begin{aligned} T'_m(\vec{v}+\vec{w}) &= (\vec{v}+\vec{w}) - T_m(\vec{v}+\vec{w}) \\ &= \vec{v}+\vec{w} - T_m(\vec{v}) - T_m(\vec{w}) \quad \text{(because } T_m \text{ is linear)} \\ &= (\vec{v} - T_m(\vec{v})) + (\vec{w} - T_m(\vec{w})) \\ &= T'_m(\vec{v}) + T'_m(\vec{w}). \end{aligned}$$

and similarly

$$\begin{aligned} T'_m(c\,\vec{v}) &= (c\,\vec{v}) - T_m(c\,\vec{v}) \\ &= c\,\vec{v} - c\,T_m(\vec{v}) \quad \text{(again since } T_m \text{ is linear)} \\ &= c\,(\vec{v} - T_m(\vec{v})) \\ &= c\,T'_m(\vec{v}). \end{aligned}$$

Therefore T'_m is also a linear transformation.

(d) Drawing a picture of $\vec{v}$, $T_m(\vec{v})$, and $\vec{v}-T_m(\vec{v})$, we see that $T'_m(\vec{v})$ is projection onto the line perpendicular to the line of slope m.



(e)

$$T'_m\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\0\end{bmatrix} - \begin{bmatrix}\frac{1}{1+m^2}\\ \frac{m}{1+m^2}\end{bmatrix} = \begin{bmatrix}\frac{m^2}{1+m^2}\\ -\frac{m}{1+m^2}\end{bmatrix}$$

$$T'_m\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}0\\1\end{bmatrix} - \begin{bmatrix}\frac{m}{1+m^2}\\ \frac{m^2}{1+m^2}\end{bmatrix} = \begin{bmatrix}-\frac{m}{1+m^2}\\ -\frac{1}{1+m^2}\end{bmatrix}$$

So the matrix for T'_m is $B_m = \begin{bmatrix}\frac{m^2}{1+m^2} & -\frac{m}{1+m^2}\\ \frac{m}{1+m^2} & -\frac{1}{1+m^2}\end{bmatrix}$.

(f) The line perpendicular to the line of slope m is the line of slope $-1/m$. Since T'_m is (by part (d)) projection onto this line, it had better be the same as $T_{(-1/m)}$, which is also projection onto the same line.

Computing $T_{(-1/m)}$, we get:

$$T_{(-1/m)} = \begin{bmatrix}\frac{1}{1+(-1/m)^2} & \frac{(-1/m)}{1+(-1/m)^2}\\ \frac{(-1/m)}{1+(-1/m)^2} & \frac{(-1/m)^2}{1+(-1/m)^2}\end{bmatrix} = \begin{bmatrix}\frac{m^2}{1+m^2} & -\frac{m}{1+m^2}\\ \frac{m}{1+m^2} & -\frac{1}{1+m^2}\end{bmatrix}.$$

(g) As $m \to \infty$ the line of slope m becomes vertical. As $m \to \infty$ the matrix A_m becomes

$$\lim_{m\to\infty} A_m = \lim_{m\to\infty}\begin{bmatrix}\frac{1}{1+m^2} & \frac{m}{1+m^2}\\ \frac{m}{1+m^2} & \frac{m^2}{1+m^2}\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$$

which is the matrix for projection onto the y axis, just as expected.