1. Put these matrices into row reduced echelon form:

(a) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 6 \end{bmatrix}$

2. Find the polynomial of degree 3 [a polynomial of the form $f(t) = a + bt + ct^2 + dt^3$] which passes through the points $(0, 1)$, $(1, 0)$, $(-1, 0)$ and $(2, -15)$.

3. Find, using Gauss-Jordan elimination, and showing all the steps, all the solutions to

$$\begin{aligned} x_1 - 7x_2 + x_5 &= 3 \\ x_3 - 2x_5 &= 2 \\ x_4 + x_5 &= 1 \end{aligned}$$

Be careful setting up the matrix correctly!

4. Solve the famous *Hundred Fowl Problem* (Prob. 60 in §2): "A rooster is worth five coins, a hen three coins, and 3 chicks one coin. With 100 coins we buy 100 of them. How many roosters, hens, and chicks can we buy?"

Let r be the number of roosters, h the number of hens, and c the number of chicks that we buy.

(a) Write down the equations specified by the problem.

(b) If r, h, and c can be arbitrary real numbers, find all solutions to the equations in part (a).

(c) If r, h, and c have to be positive integers (i.e., whole numbers), and c has to be a multiple of 3, find all the solutions.

5. For what values of k and ℓ does the augmented system

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 2 & k & 1 & 2 \\ 1 & 10 & k & \ell \end{array}\right]$$

have

(a) Exactly one solution?

(b) No solutions?

(c) Infinitely many solutions?

Find all the solutions in case (c).