DUE DATE: DEC. 7, 2004

1. For the basis $\beta = (3,5),(1,2)$ in $\mathbb{R}^2$,

- Express $(4,3)$, $(1,2)$, and $(1,3)$ in β coordinates.
- Express $(2,0)_\beta$, $(1,1)_\beta$, and $(-1,4)_\beta$ in the standard coordinates.

2. Suppose that $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ is given (in the usual coordinates) by the matrix

$$A = \begin{bmatrix} 2 & 1 & 5 \\ 1 & 1 & 3 \end{bmatrix}.$$

If $\beta = (1,1,1)$, $(2,0,1)$, and $(3,2,1)$ is a new basis in $\mathbb{R}^3$, and $\alpha = (3,5)$, $(1,2)$ a new basis in $\mathbb{R}^2$, find the matrix for T with respect to the new basis on both sides.

3. Suppose that $v_1 = (1,1)$, $v_2 = (1,-1)$, and that the basis β is $\beta = v_1, v_2$.

Let T be the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ given by $T(v_1) = v_1$ and $T(v_2) = \vec{0}$.

(a) Write down the matrix for β in the new basis β. (You should be able to do this directly from the definition of T).

(b) Use this to write down the matrix for T in the standard basis.

You might want to compare the answer for (b) with the answer for homework 3, question 4, with $m = 1$. Can you see why these are the same?

4. Let V be the subspace of the vector space of all functions spanned by $\sin(x)$ and $\cos(x)$, i.e., functions of the form $a\sin(x) + b\cos(x)$. The functions $\sin(x)$ and $\cos(x)$ are a basis for V.

The map $T : V \longrightarrow V$ given by $T(f) = f'$ (i.e., f is mapped to its derivative) is a linear transformation from V to V. Write its matrix in the $\sin(x)$, $\cos(x)$ basis.