DUE DATE: SEPT. 28, 2004

1. Compute each expression

(a) $\begin{bmatrix} 3 & 5 & 8 \\ 1 & 2 & -4 \end{bmatrix} + \begin{bmatrix} 4 & 6 & 0 \\ 0 & -3 & 2 \end{bmatrix}$ (b) $3\begin{bmatrix} 1 & -2 \\ -6 & 5 \end{bmatrix}$

(c) $4\begin{bmatrix} 1 & 0 & 4 \\ 2 & 7 & 3 \end{bmatrix} - 3\begin{bmatrix} 2 & 1 & 1 \\ 7 & 0 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -2 & 5 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

2. For parts (a), (b), and (c), answer following questions:

Is $\vec{b}$ a linear combination of $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$?
If so, find all the possible ways to express $\vec{b}$ as a linear combination of those three vectors.

(a) $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$, and $\vec{b} = \begin{bmatrix} 1 \\ 3 \\ 10 \end{bmatrix}$.

(b) $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 0 \\ 2 \\ -4 \end{bmatrix}$, and $\vec{b} = \begin{bmatrix} 1 \\ 3 \\ 10 \end{bmatrix}$.

(c) $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 0 \\ 2 \\ -4 \end{bmatrix}$, and $\vec{b} = \begin{bmatrix} 8 \\ 5 \\ 6 \end{bmatrix}$.

3. Let A be a 4×3 matrix, and let $\vec{b}$ and $\vec{c}$ be two vectors in $\mathbb{R}^4$. We are told that the system $A\vec{x} = \vec{b}$ has a unique solution. What can you say about the number of solutions of the system $A\vec{x} = \vec{c}$?

4. Suppose that $\vec{v}_1, \ldots, \vec{v}_4$ are vectors in $\mathbb{R}^3$. Form the matrix

$$A = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 & \vec{v}_4 \end{bmatrix}$$

whose columns are the vectors $\vec{v}_1, \ldots \vec{v}_4$. Suppose that

$$\mathrm{RREF}(A) = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Is $\vec{v}_3$ a linear combination of $\vec{v}_1$ and $\vec{v}_2$? If so, with what coefficients?

(b) Is $\vec{v}_4$ a linear combination of $\vec{v}_1$ and $\vec{v}_3$? If so, with what coefficients?

(c) Is $\vec{v}_3$ a linear combination of $\vec{v}_1$ and $\vec{v}_4$? If so, with what coefficients?

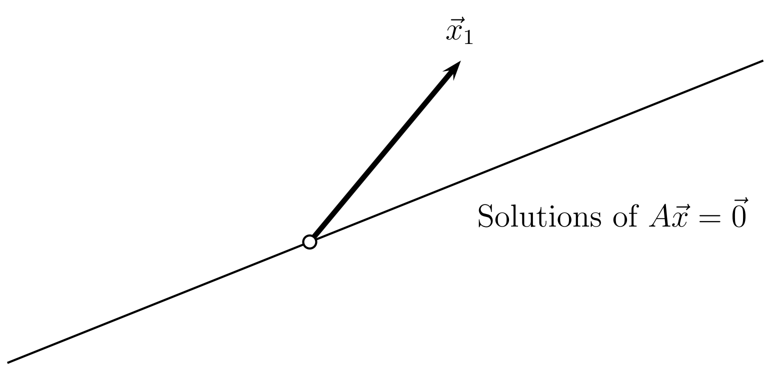
What is the principle that allows you to answer these questions? i.e., you don't know what the vectors $\vec{v}_1, \ldots \vec{v}_4$ are, but it's still possible to give exact answers to all these questions. Why?

5. (Problem 48 from §1.3) Consider a solution $\vec{x}_1$ of the linear system $A\vec{x}_1 = \vec{b}$. Justify the facts stated in parts (a) and (b):

(a) If $\vec{x}_h$ is a solution of the system $A\vec{x}_h = \vec{0}$, then $\vec{x}_1 + \vec{x}_h$ is a solution of the system $A\vec{x} = b$.

(b) If $\vec{x}_2$ is another solution of the system $A\vec{x} = \vec{b}$, then $\vec{x}_2 - \vec{x}_1$ is a solution of the system $A\vec{x} = 0$.

(c) Now suppose that A is a 2×2 matrix. A solution vector $\vec{x}_1$ of the system $A\vec{x} = \vec{b}$ is shown in the figure below.



We are told that the solutions of the system $A\vec{x} = \vec{0}$ form the line shown in the sketch. Draw the line consisting of all solutions of the system $A\vec{x} = \vec{b}$.

If the problem seems too general, it might help to think of a concrete example like

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 3 \\ 9 \end{bmatrix}, \quad \text{and } \vec{x}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$