DUE DATE: OCT. 13, 2004

1. Compute these matrix multiplications:

(a) $\begin{bmatrix} 1 & -2 \\ 3 & 1 \\ 4 & -7 \end{bmatrix} \begin{bmatrix} 1 & -3 & -3 \\ 2 & 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -3 & -3 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 3 & 1 \\ 4 & -7 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 3 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ 1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & 3 \\ -2 & -3 & 1 \\ 4 & 5 & 2 \end{bmatrix} \begin{bmatrix} 4 & 2 & 1 \\ 3 & 3 & 3 \\ 0 & 1 & 2 \end{bmatrix}$

(e) $\left[\begin{array}{cc|cc} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{cc|cc} 1 & 2 & 2 & 3 \\ 3 & 4 & 4 & 5 \\ \hline 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 4 \end{array}\right]$

It may be easier to compute (e) by using block matrix multiplication.

2. Suppose we have two linear transformations $T_1 : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ and $T_2 : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ given by these formulas:

$$T_1(x, y, z) = \begin{bmatrix} 3x + 7z \\ x + 2y + 8z \end{bmatrix}, \text{ and } T_2(x, y) = \begin{bmatrix} x + 4y \\ 2x + 3y \\ -x + 2y \end{bmatrix}$$

(a) Give the formulas for the composite function $T_3 = T_2 \circ T_1$.

(b) Using these formulas, find the matrix C for T_3.

(c) Find the matrix A for T_1 and B for T_2.

(d) Compute the matrix product BA showing the details of how you computed the entries. (You should, of course, get matrix C as an answer.)

3. Determine if the linear transformations described by the following matrices are invertible. If not, explain why, and if so, find the matrix of the inverse transformation.

(a) $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 & 6 \\ 0 & 3 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & 7 \\ 3 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 1 \\ 10 & 5 \end{bmatrix}$

(e) $\begin{bmatrix} 2 & 1 & 6 \\ 4 & 3 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ (f) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 5 & 6 & 1 & 0 \\ 7 & 10 & 4 & 1 \end{bmatrix}$

4. Suppose that A is the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 1 & 7 \\ 2 & 2 & 1 \end{bmatrix}.$$

(a) Find the inverse of A.

(b) Explain why, for any values of a, b, and c, the equations

$$\begin{array}{rcrcrcl} x & + & 2y & + & 3z & = & a \\ -2x & + & y & + & 7z & = & b \\ 2x & + & 2y & + & 1z & = & c \end{array}$$

always have a unique solution.

(c) Find this unique solution (in terms of a, b, and c).

5.

(a) Find the RREF of the matrix $\left[\begin{array}{cc|ccc} 9 & 5 & 1 & 3 & 2 \\ 5 & 3 & 1 & 1 & 3 \end{array}\right]$.

(You can ignore the dots, I just felt like putting them in.)

(b) If A is the matrix $A = \begin{bmatrix} 9 & 5 \\ 5 & 3 \end{bmatrix}$, find solutions to the linear equations $A\vec{x} = \vec{b}$ for the vectors $\vec{b}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\vec{b}_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, and $\vec{b}_3 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (i.e., you should find solutions to these three different systems of equations).

(c) If B is the inverse matrix for A, with column vectors $\vec{v}_1$ and $\vec{v}_2$, (so $B = \left[\vec{v}_1 \,\vdots\, \vec{v}_2\right]$), explain why $A\vec{v}_1 = \vec{e}_1$ and $A\vec{v}_2 = \vec{e}_2$. The kind of thinkng I had in mind was this: B is supposed to be the matrix for the transformation that's inverse to the transformation for A, and maybe thinking about what the inverse is supposed to do, and how you figure out the matrix for a linear transformation would lead to an explanation.

(d) Explain why putting the matrix $\left[\begin{array}{cc|cc} 9 & 5 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{array}\right]$ into RREF gives you $\left[I_2 \,\vdots\, B\right]$.

That is, why does the second matrix have to be B, and not any other matrix? I guess I'm really asking: why does this method of computing the inverse work?