

This homework assignment has only one question, and that's because it's a very good question. It's also a challenging question: it shouldn't be immediately obvious what you should do to solve it, and it will require some playing around with examples, false starts, going around in circles, and finally an idea or two to get it going.

Even when you've figured out a strategy to solve the problem, it will take some care and organization to work out the details, and even more care to write them down in a clear way.

I'd like to see what you can do with a problem of this kind: Something new that you haven't seen before, but something within your powers to solve.

Ready? Here it is:

For any positive integer n , let a_n be the number of ways that you can write n as a sum using only the numbers 1, 2, and 3, where the order of the sum doesn't matter.

For example, $a_1 = 1$, since the only possible "sum" is $1 = 1$, and $a_2 = 2$ since we can write 2 as a sum in two ways: $2 = 1 + 1$ and $2 = 2$. Next $a_3 = 3$ since we can write $3 = 1 + 1 + 1$, $3 = 1 + 2$, and $3 = 3$. Notice that we consider $1 + 2$ and $2 + 1$ to be the same sum – we don't care about the order, it's just the number of 1's, 2's, and 3's that matter.

Here's a table of the first four a_n 's and the ways to write each n as such a sum:

n	a_n	ways
1	1	1
2	2	1 + 1, 2
3	3	1 + 1 + 1, 1 + 2, 3
4	4	1 + 1 + 1 + 1, 1 + 1 + 2, 2 + 2, 1 + 3

I'd like to know a way to compute a_n without writing out all the possibilities. To make it more concrete I'd like to know what the values of a_n are for $n = 2001, 2002, 2003, 2004, 2005$, and 2006.

So that's your problem: Compute a_n for these six values of n .

I wouldn't try and work out a_n just for these six values, that seems too difficult. Instead I'd try and work out a way of calculating a_n for any n , and apply the method to these values of n .

How could you do that? I guess I'd start by working out some more values of a_n , to try and get an idea of what could happen. Then maybe I'd try and figure out a way to organize the counting a bit more systematically, and maybe even get a recursive way of computing a_n .

Without trying to be misleading, I should say that there is actually a general formula for a_n , or more accurately general formulas. Which formula you use depends on what kind of number n is. (For some of the values of n you use one formula, for some values of n another formula.) One possible solution to the problem is to find those formulas, although there are certainly other ways.

If none of that makes sense, that's okay, you don't have to use any particular method to plan to solve the problem, any way that you find to solve the problem is okay.

So – give it a try. Don't be discouraged if it seems confusing at first, that's the nature of interesting problems, they don't always yield to the first attempt, but are all the more rewarding for the time it takes to unravel them.