

# Mathematical Modeling & Dynamics with Delay

June 10–12, 2026  
Chernoff Hall, Queen's University  
Kingston, ON, Canada

In honour of the 60th birthday of Professor Tony Humphries (McGill University)

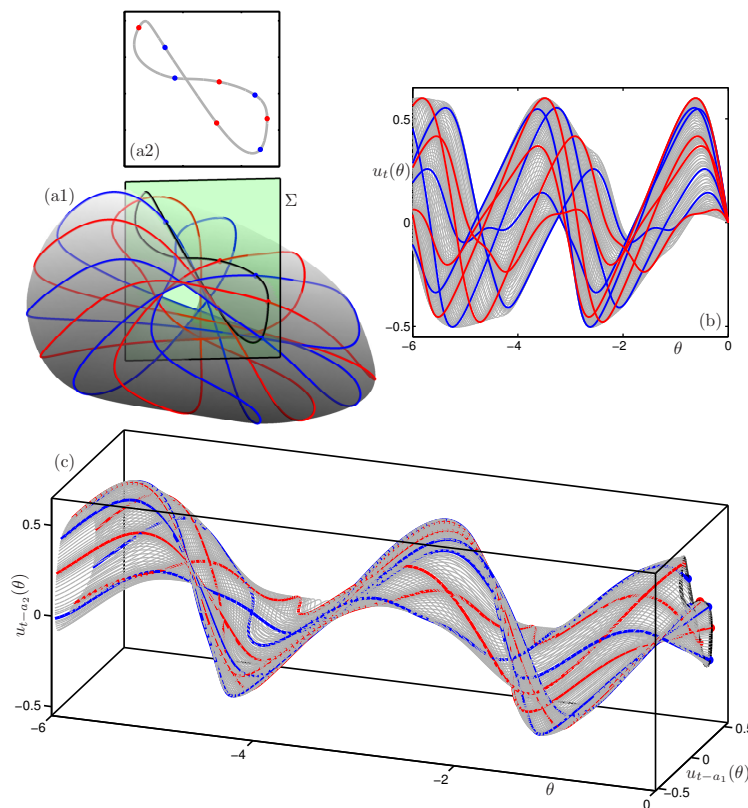


Image from R. C. Calleja, A. R. Humphries, and B. Krauskopf, Resonance phenomena in a scalar delay differential equation with two state-dependent delays, *SIAM J. Appl. Dyn. Syst.*, 16(3), 1474–1513, 2017.

This workshop is funded by the Canadian Applied and Industrial Mathematics Society (Dynamical Systems Activity Group), Centre de recherches mathématiques (Laboratoire Mathématiques appliquées) and the Society for Mathematical Biology.

## Organizers

- Felicia Magpantay (Queen's University)
- Tyler Cassidy (University of Leeds / University of British Columbia)
- Shaza Alsibai (Queen's University)

## Plenary Speakers

- Dimitri Breda (Università degli Studi di Udine)
- Renato Calleja (Universidad Nacional Autónoma de México)
- Sue Ann Campbell (University of Waterloo)
- Morgan Craig (Université de Montréal)
- Bernd Krauskopf (University of Auckland)

## Speakers

- Shaza Alsibai (Queen's University)
- Jacques Bélair (Université de Montréal)
- Tyler Cassidy (University of Leeds / University of British Columbia)
- Alexey Eremin (Saint Petersburg State University) [video]
- Nicola Guglielmi (Gran Sasso Science Institute) [video]
- Tony Humphries (McGill University)
- Jean-Philippe Lessard (McGill University)
- Hinke Osinga (University of Auckland)
- Stefan Ruschel (University of Nottingham)
- Francesca Scarabel (University of Leeds)
- Jan Sieber (University of Exeter)
- Rossana Vermiglio (Università degli Studi di Udine) [video]
- Gail Wolkowicz (McMaster University)

## Programme Schedule

All talks take place in Chernoff Hall Room 213. Coffee breaks are in Chernoff Hall Room 211. Lunch (June 10–12) is at Ban Righ Hall.

### Wednesday, June 10

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|             |                            |
|-------------|----------------------------|
| 8:30–9:00   | Coffee and welcome remarks |
| 9:00–10:00  | Morgan Craig               |
| 10:00–10:30 | Coffee Break               |
| 10:30–11:00 | Tyler Cassidy              |
| 11:00–11:30 | Gail Wolkowicz             |
| 11:30–12:00 | Shaza Alsibaa              |
| 12:00–13:30 | Lunch                      |
| 13:30–14:30 | Bernd Krauskopf            |
| 14:30–15:00 | Coffee Break               |
| 15:00–17:00 | Early Career Session       |

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### Thursday, June 11

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|-------------|--------------------|
| 9:00–10:00  | Sue Ann Campbell   |
| 10:00–10:30 | Coffee Break       |
| 10:30–11:00 | Jan Sieber         |
| 11:00–11:30 | Francesca Scarabel |
| 11:30–12:00 | Jacques Bélair     |
| 12:00–13:30 | Lunch              |
| 13:30–14:30 | Renato Calleja     |
| 14:30–15:00 | Coffee Break       |
| 15:00–15:30 | Stefan Ruschel     |
| 15:30–16:00 | Rossana Vermiglio  |
| 16:00–16:30 | Hinke Osinga       |
| 16:30–17:00 | Tony Humphries     |

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### Friday, June 12

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|-------------|-----------------------|
| 9:00–10:00  | Dimitri Breda         |
| 10:00–10:30 | Coffee Break          |
| 10:30–11:00 | Jean-Philippe Lessard |
| 11:00–11:30 | Nicola Guglielmi      |
| 11:30–12:00 | Alexey Eremin         |
| 12:00–13:30 | Lunch                 |

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**Wednesday, June 10**

**Morgan Craig (Université de Montréal)**

*Mechanisms of immune regulation in response to viral infections and vaccines in healthy and immunocompromised hosts*

The immune system is regulated by multiple nonlinear feedback loops, resulting in complex dynamical immune responses. Understanding the regulation of immunity is critical to identifying populations more at-risk of severe disease after infection and tailoring vaccination schedules to boost protection in vulnerable people. In this talk, I will discuss our work to unravel the mechanisms of immune regulation in response to viral infections and vaccines in healthy and immunocompromised hosts using mechanistic mathematical models. Owing to the complexity of the immune system, these models integrate delays to account for cellular and cytokine stimulation and inhibition. I will discuss how we use our models to generate virtual patient cohorts and run virtual clinical trial, ultimately identifying how multiple factors, including age and comorbidities, impact the immune response to pathogens and vaccines.

**Tyler Cassidy (University of Leeds)**

*Efficient simulation and inference in delayed biological models*

Delayed processes are ubiquitous throughout biology. Delay differential equations represent a natural modelling framework to capture these delayed dynamics. However, these delay differential equations can be difficult to simulate efficiently, which complicates parameter inference. Here, I'll discuss recent work that facilitates efficient model simulation and parameter inference without imposing artificial constraints on biological processes, and I'll illustrate the computational benefits of these approaches through a series of examples from across mathematical medicine.

**Gail Wolkowicz (McMaster University)**

*A predator-prey model with delay in both the prey and predator growth terms*

Previous research on a two-species Gause type predator-prey model with mass action interaction between the prey and the predator showed that the introduction of delay in the predator growth term can result in exotic dynamics in the form of stable periodic orbits with large or small amplitude, and a cascade of period doubling bifurcations leading to chaos. In this thesis, we formulate a predator-prey model with delay in the growth terms of both the predator and the prey. We assume that each population must survive the delay period, consistent with the decay terms in the model, in order to contribute to the growth of their respective populations. We use analytical methods to establish conditions on the effect of delay in the prey growth term on the existence and stability of equilibria, as well as the persistence of the prey population, with and without delay in the predator population growth term. In addition, we use numerical methods to show the effects of delay in the prey growth term on dynamics of the model with delay in the predator growth term. Short delays in the prey growth term can stabilize the interior equilibrium in regions displaying oscillations caused by delay in the predator growth term. Delay in the prey growth term can reduce the amplitude of periodic orbits, and trigger period-halving bifurcations in chaotic regions, ultimately stabilizing the interior equilibrium. Longer delays in the prey growth term can result in the extinction of the predator

population. Even longer delays in the prey growth term are shown to decrease the size of the prey population and lead to its extinction. This study extends previous findings on the stabilizing effect of delay in the prey growth term on oscillatory predator-prey systems to a new class of models.

Joint work with Konstantin Lenz-Gil and Christopher Greyson-Gaito

### **Shaza Alsibai (Queen's University)**

#### *Mathematical modelling of erythropoiesis and iron metabolism of blood donors*

Several experimental studies have shown that iron deficiency is common among regular blood donors. Indeed, the recommended inter-donation interval in the United States and Canada is based on old studies investigating only hemoglobin recovery after blood donation. Recent experimental studies, which measure other iron parameters after blood donation, have shown that this interval is very short to prevent iron deficiency in frequent blood donors. In this talk, I will present a mathematical model for erythropoiesis and iron metabolism and apply it to this problem. I will start with a simple mathematical model of erythropoiesis that captures the key physiological mechanisms governing the process. Then, I will present a combined model of erythropoiesis and iron metabolism to study the impact of repeated blood donations on iron levels in regular blood donors. This model consists of seven coupled delay differential equations (DDEs) with both discrete and distributed delays. The use of distributed delays allows for a more accurate representation of hormonal effects on erythroid precursor cells. I will demonstrate how to reformulate the distributed DDEs into a system of discrete DDEs, which we can solve using available packages in MATLAB. Through numerical simulations, I will illustrate the effects of single and repeated blood donations on hemoglobin and storage iron. I will also show the impact of extending the inter-donation interval and administering iron supplementation on hemoglobin and storage iron recovery. Finally, I will discuss the significant influence of inadequate dietary iron intake, particularly for frequent blood donors.

### **Bernd Krauskopf (University of Auckland)**

#### *Resonances in DDE climate models: a conceptual view of El Niño*

Feedback loops in climate systems arise from interactions between various subsystems, such as distinct bodies of water, the atmosphere, land and ice masses. Importantly, each feedback takes effect only after an inherent time delay. Such delayed feedback loops give rise to conceptual models in the form of DDEs. We consider here as a concrete example a conceptual DDE model for the El Niño Southern Oscillation phenomenon (ENSO). This DDE describes the interplay between a negative feedback loop and seasonal forcing. We show how El Niño events arise from 'imperfect locking' due to resonances between forcing and delay. We also discuss when and how state-dependence of the delay may play an important role for the observed resonance structure.

This is joint work with Andrew Keane (University College Cork), Claire Postlethwaite (University of Auckland) and Henk Dijkstra (Utrecht University).

## Early Career Session

### Samuel Bolduc-St-Aubin (University of Auckland)

#### *From resonance to chaos in a delayed-feedback model of El Niño-Southern Oscillation*

The El Niño–Southern Oscillation (ENSO) is responsible for irregular warming and cooling episodes in the equatorial Pacific Ocean. ENSO models come at many levels of complexity. We focus on conceptual ENSO models, which condense the essential physics into manageable mathematical forms to facilitate intuition-building and analysis with advanced techniques. The delayed-action oscillator (DAO) paradigm explains El Niño events as arising from a time-delayed negative feedback loop driven by the propagation of large-scale oceanic waves. Motivated by the observation that ENSO events tend to lock to the seasonal cycle, we consider a periodically forced version of the very first DAO model of ENSO due to Suarez and Schopf.

We present a detailed bifurcation analysis of this periodically forced delay differential equation. Due to the infinite-dimensional nature of DDEs, this ENSO model features a rich range of dynamical behaviors, including flows on invariant tori and chaotic dynamics. By varying the forcing frequency, we show that the observed chaotic dynamics arise through period-doubling cascades and overlapping resonance tongues. Using a new method to compute the rotation number on stable invariant tori for DDEs, we examine precisely how this overlap unfolds.

### Catherine Engert (University of Oxford)

#### *Mathematical modelling of immune system dynamics during flu infections*

Simple ordinary differential equation virus dynamics models can be used to describe within-host infection time courses. These typically assume that the immune response is constant over time and fails to account for the components of the immune response not being activated at the same time, due to many factors, including pre-existing immunity. To understand the evolution of long-term immunity and the impact of recurrent infections, we developed a delay differential equation model that explicitly accounts for both the innate and adaptive immune responses, which are broadly the first and second phases of the immune response, and the generation of immunological memory post-infection. In this talk, I will present the underlying mechanisms of the immune response, from which we derive our model. Numerical simulations of the model, under different conditions, will be presented to illustrate how this model can be used to understand immunity dynamics and study strategies that enhance immunity and minimize infection and reinfection.

### Gabriel Côté (Université de Montreal)

#### *Characterizing neutrophil circadian rhythms to optimize neutrophil-targeting therapies in lung cancer*

Oscillations, particularly circadian rhythms, are ubiquitous in physiology and there is a growing recognition that these rhythms impact the administration of treatments and disease prognostics. For instance, neutrophils, the most abundant immune cells in blood, display circadian oscillations under the control of the CXCR2 and CXCR4 receptors. In lung cancer, CXCR2 inhibitors were suggested to reduce neutrophil infiltration into the tumour, as this has been shown to promote metastases. However, murine experimental evidence showed that administration of CXCR2 inhibitors must align with circadian rhythms; when timed improperly these inhibitors were found to have little to no effect.

Even worse, improper timing could result in the dangerous depletion of neutrophils, leading to worst outcomes for the patient. Thus, there is a need to rationalize CXCR2 inhibitor treatment schedules. We developed a mathematical model using state-dependent delay differential equations (SD-DDE) with periodic forcing to describe neutrophil production. Our model incorporates CXCR2 regulation and circadian inputs. We distinguished how key circadian mechanisms regulating neutrophil counts in the blood impact regulators of granulopoiesis (e.g., G-CSF, CXCR2 and CXCR4) on neutrophil circadian rhythms. Our findings also illustrate how periodic forcing may be analyzed in SD-DDE models. This study underlines the importance of chronobiology to drug and immune responses. Our work may be extended to investigate immunity in shift workers, jet-lagged travellers, and individuals with circadian rhythm sleep disorders.

### **Kamyar Tavakoli (University of Ottawa)**

#### *Multiple delays in nonlinear dynamical systems*

Delay differential equations with multiple delays provide a natural framework for modeling real world systems in fields such as neuroscience, physics, and engineering. The distribution of delays plays an important role in determining dynamical and statistical properties. In some cases, introducing even a small number of delays can make the dynamics more intricate and increase complexity, while in others it suppresses complexity and leads to simpler dynamics. These features make multi-delay systems well-suited to a wide range of contexts, from natural systems to computational frameworks such as time-delay reservoir computing, where the structure of delays affects memory capacity and computational performance. In this talk, I will discuss how the delay distribution affects the complexity of multi-delay dynamics, and how these ideas can be useful in problems such as modeling seizure dynamics, time series prediction, and signal demixing.

### **Tianyu Cheng (York University)**

#### *Global dynamics of a differential system coupled with both discrete and distributed delays*

In this work, we consider a differential system with unimodal feedback that includes both discrete and distributed delays, each with its own weight. This model represents the tick population with both discrete and distributed delays for normal development delay and diapause-induced time delay, respectively. We derive threshold conditions for the occurrence of local Hopf bifurcations and theoretically study their global continuation.

**Thursday, June 11**

**Sue Ann Campbell (University of Waterloo)**

*Time delays, Hopf bifurcation and synchronization*

We consider networks of oscillator nodes with time delayed, global circulant coupling. We first study the existence of Hopf bifurcations induced by coupling time delay and then use symmetric Hopf bifurcation theory to determine how these bifurcations lead to different patterns of phase-locked oscillations. We apply the theory to a variety of systems inspired by biological neural networks to show how Hopf bifurcations can determine the synchronization state of the network. We show how the network structure can influence the existence of co-dimension two double Hopf bifurcations which in turn affects whether the nodes synchronize. We consider both discrete and distributed delays and determine how the type of distribution and size of the mean delay affect the results.

**Jan Sieber (University of Exeter)**

*Neutral delay differential equations with state-dependent delays*

We will show which results and numerical methods can still be applied when studying neutral delay differential equations where state-dependent delays occur in the neutral terms (so the delayed derivatives depend on state-dependent delays). In particular, the Hopf bifurcation theorem still holds, when one imposes a smallness condition on the delayed derivatives that is compatible with what one can expect for state-dependent delays.

This is joint work with Ahlam Alhadbani.

**Francesca Scarabel (University of Leeds)**

*Investigating multistability and basins of attraction in models with time delay*

In this talk I will present a recent approach to investigate multistability and the basins of attraction of dynamical systems with time delays. First, an approach to sample initial conditions and automatically detect the attractors of the system is proposed. Then, statistical dimensionality reduction techniques are used to extract important features of the initial conditions to interpret and visualise the basins of attractions of the system. The technique is illustrated using an infectious disease model with waning and boosting of immunity that admits multistability and other complex behaviours [1]. This work is part of an ongoing project with Evert Provoost (KU Leuven).

[1] Scarabel, F., Polner, M., Wylde, D., Barbarossa, M.V., and Röst, G. (2025) Bistability and complex bifurcation diagrams generated by waning and boosting of immunity, *Journal of Mathematical Biology*, 91:30. <https://doi.org/10.1007/s00285-025-02264-3>

**Jacques Bélair (Université de Montréal)**

*Coupled delayed negative feedback loops in biological systems: oscillations and stability*

Biological regulatory systems are often appropriately modeled as coupled systems of nonlinear delay differential equations. For example, the production of mammalian blood cells involves an inter-

twined network of physiological processes, with nonlinear, delayed feedback control mechanisms: erythrocytes (red blood cells) and thrombocytes (platelets) while each having their own main regulatory hormone, erythropoietin and thrombopoietin respectively, are also interacting, specially in pathological conditions. We consider the (highly) simplified model

$$\begin{aligned}x'(t) &= -\alpha x(t) + f(x(t - \tau), y(t\tau)) \\y'(t) &= -\beta y(t) + g(x(t - \tau), y(t - \tau))\end{aligned}$$

with  $f$  and  $g$  appropriate Hill functions for the coupled regulation of these two cell lines to study how the interaction of the control mechanisms may influence the dynamics. Equilibrium solutions are determined, their stability established and the nature of the oscillations when instability occurs are investigated. The linear analysis revolves around a transcendental characteristic equation of second order with two delays; a Centre manifold analysis at the change of stability of equilibria provides insight into possible dynamics. Possible extensions will also be discussed.

**Renato Calleja (Universidad Nacional Autónoma de México)**

*Resonance phenomena in delay differential equations with two state-dependent delays*

State-dependent delay differential equations arise naturally in applications where the delay is influenced by the current state of the system, leading to rich and often highly nontrivial dynamical behavior. In this talk, I will revisit joint work with Tony Humphries and Bernd Krauskopf on a scalar delay differential equation with two state-dependent delays, originally motivated by resonance phenomena and nonlinear oscillations. The equation exhibits intricate interactions between the delays and the oscillatory dynamics, including resonance tongues, phase-locking structures, and transitions between qualitatively different regimes. A central aspect of the analysis is the geometric organization of periodic solutions and bifurcations in parameter space, together with the numerical continuation techniques required to study such systems. Beyond the specific model, the work illustrates broader challenges and opportunities in the study of state-dependent delays, where analytical questions, numerical methods, and dynamical systems theory interact in subtle ways. I will also reflect on the influence that Tony Humphries has had on this area and on the collaboration that led to these results.

**Stefan Ruschel (University of Nottingham)**

*Network dynamics driven by adaptive delays*

We will present on frequency selection and network attractor formation in weakly nonlinear coupled systems by means of delay plasticity. Specifically, we consider adaptive axonal delays (AADs), motivated by activity-dependent myelination in the brain, which regulate signal propagation speeds and thus communication delays. We demonstrate frequency selection and network attractor formation in systems of delay-coupled phase oscillators with two prototypical choices of AADs on brain connectivity data and globally coupled ring networks.

**Rossana Vermiglio (Università degli Studi di Udine)**

*Sun–Star calculus and numerical methods for delay equations*

The sun–star calculus, in the sun-reflexive setting, provides an effective mathematical framework for the analysis of delay equations (DEs), including delay differential equations (DDEs), renewal equations (REs), and coupled RE–DDE systems. By reformulating these equations as semilinear abstract evolution equations and deriving the variation-of-constants formula, a unified framework for the development of efficient numerical methods can be obtained. This approach supports stability and bifurcation analysis, as well as time integration. The talk presented an overview of this class of numerical methods, including exponential-type time-integration schemes. It also addressed extensions to delay equations with infinite delay, discussed applications to structured population models, and outlined directions for future research.

**Hinke Osinga (University of Auckland)**

*Phase resetting in a system of coupled Van der Pol oscillators*

Synchronisation properties of coupled nonlinear oscillators can be probed by studying the response to external perturbations: after relaxation back to the stable oscillation, there is generally a phase shift. Important information can be gained by studying such phase resets as a function of when the perturbation is applied during the oscillation. Phase resetting is a common tool in experiments on neurons. We developed a versatile and efficient numerical method for computing phase resetting curves for systems of any dimension; our approach is based on continuation of solutions to a multi-point boundary value problem. We present a case study of a prototypical example: two coupled 1:1 phase-locked Van der Pol oscillators. In contrast to single oscillators, this system has a phase space of dimension four. In particular, the basin of attraction of the stable synchronised oscillation has a complicated boundary, and we show how this affects the observed phase resetting in unexpected ways.

This is joint work with Kyoung Hyun Lee, Neil Broderick and Bernd Krauskopf.

**Tony Humphries (McGill University)**

*A journey into delays*

Threshold delays arise naturally in a wide variety of dynamical systems, including maturation and transport processes, and can be found in models of hematopoiesis, cell biology and immunology to name just a few examples. We'll finish this talk by discussing how to deal with them, and some of the issues that arise. But, with so many former students and collaborators and colleagues present, I'll spend most of this talk describing my own transport process from pure mathematics into numerical analysis and dynamical systems, then through advanced-retarded equations into delay equations and finally mathematical biology, and in particular some of the people that influenced me along the way.

**Friday, June 12**

**Dimitri Breda (Università degli Studi di Udine)**

*On biological cascades and differential equations with advances*

In natural and synthetic gene networks, Incoherent Feed Forward Loops (IFFLs) are known to generate pulses, accelerate response times, and filter input signals. While the behavior of a single IFFL has been extensively studied, far less is understood about circuits that include IFFL interconnections. A recent study on IFFLs interconnected in a cascade, a topology of potential interest for generating controllable temporal or spatial patterns in engineered or synthetic cells, shows how a pulsatile signal emerges in response to given inputs. The features of the pulse appear to converge to a coherent signal traveling along the cascade. A conjecture is discussed, suggesting that the asymptotic profile of the infinite IFFL cascade is characterized by a correlated differential equation with advance. This connection would help unravel the mechanism by which, along the cascade, the input to an IFFL is an advance version of its output.

This is a joint work with E. Franco, E. Nakamura and B. Whitley from UCLA and F. Blanchini from the University of Udine.

**Jean-Philippe Lessard (McGill University)**

*Computer-assisted and constructive methods for periodic orbits in differential equations with large state-dependent delays*

In this talk, we present a constructive, computer-assisted framework for proving the existence of periodic solutions in SDDEs with large state-dependent delays. The approach combines Fourier expansions with a Newton–Kantorovich argument in a low-regularity Banach space, together with rigorous treatment of composition operators via discrete Fourier analysis, contour integrals, and the discrete Poisson formula. We illustrate the method through three applications of increasing complexity: (1) a forced SDDE with prescribed period; (2) an autonomous SDDE where the period is part of the unknowns, following Calleja, Humphries, and Krauskopf (2018); and (3) an SDDE with a threshold nonlinearity, requiring unfolding parameters to handle the discontinuity and determine the period, based on Gedeon, Humphries, Mackey, Walther, and Wang (2025).

This is joint work with Jan Bouwe van den Berg (VU University Amsterdam), Maxime Breden (École Polytechnique), Matthieu Cadiot (École Polytechnique), and Kevin Church (CRM, Montréal).

**Nicola Guglielmi (Gran Sasso Science Institute)**

*Solving time-fractional differential equations*

A fractional differential equation is a generalization of an integer-order differential equation, where the (local) integer derivative operator is replaced by a (nonlocal) fractional derivative. With the help of the Riemann–Liouville fractional integral, such problems can be transformed into Volterra integro-differential equations. The main idea of our numerical approach is to approximate the fractional kernel by a sum of exponential functions, and then to transform the fractional integral (of convolution type) into a set of ordinary differential equations. Solving the resulting (stiff) system by a numerical time integrator leads to a fast and memory-efficient method. We also explain how

the code RADAU5 can be used to solve fractional differential equations. Numerical experiments illustrate the accuracy and efficiency of the proposed approach.

This is joint work with Ernst Hairer (Université de Genève).

**Alexey Eremin (Saint Petersburg State University)**

*Continuous Runge–Kutta Methods with Embedded Handling of Overlapping*

Continuous Runge–Kutta methods (CRKs) for state-dependent delay differential equations can meet troubles when the delay is too small and the delayed solution is required within the current step. Practically this leads to implicit implementation of explicit methods (possibly made explicit by iterative recomputation of the step). Another approach is to develop Functional Continuous Runge–Kutta methods (FCRKs) — fully explicit methods with continuous stages. For methods of order 4 and higher Functional Continuous methods require more stages than usual Continuous methods. Typically, overlapping occurs not so often, and it would be possible to take a CRK and an FCRK pair, but it can also be too expensive if the CRK finds out overlapping at one of the last stages and needs to discard the previously made computations. We present embedded pairs, which checks every stage independently and can save the computations if there is no overlapping, or recompute the stage to have a necessary stage order if there is overlapping. The 4th order method makes 6 stages as a CRK and 7 as an FCRK if necessary. The 5th order method makes 8 stages as a CRK and makes up to 3 additional computations if necessary being a full FCRK in the worst case. The last stage reuse is incorporated into all the methods.