

Mathematical Modeling & Dynamics with Delay

June 10–12, 2026
Chernoff Hall, Queen's University
Kingston, ON, Canada

In honour of the 60th birthday of Professor Tony Humphries (McGill University)

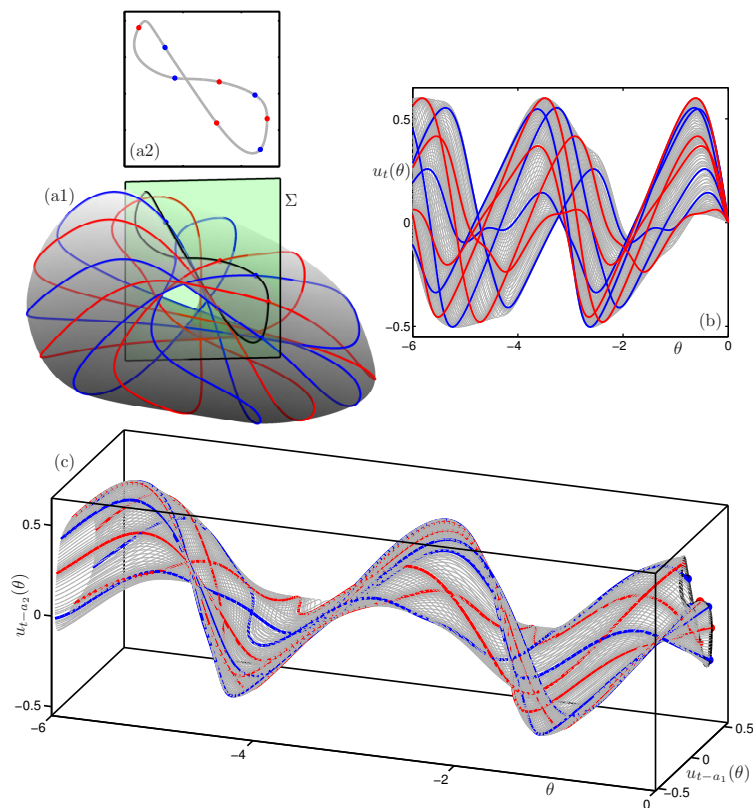


Image from R. C. Calleja, A. R. Humphries, and B. Krauskopf, Resonance phenomena in a scalar delay differential equation with two state-dependent delays, *SIAM J. Appl. Dyn. Syst.*, 16(3), 1474–1513, 2017.

This workshop is funded by the Canadian Applied and Industrial Mathematics Society (Dynamical Systems Activity Group), Centre de recherches mathématiques (Laboratoire Mathématiques appliquées) and the Society for Mathematical Biology.

Organizers

- Felicia Magpantay (Queen's University)
- Tyler Cassidy (University of Leeds / University of British Columbia)
- Shaza Alsibai (Queen's University)

Plenary Speakers

- Dimitri Breda (Università degli Studi di Udine)
- Renato Calleja (Universidad Nacional Autónoma de México)
- Sue Ann Campbell (University of Waterloo)
- Morgan Craig (Université de Montréal)
- Bernd Krauskopf (University of Auckland)

Speakers

- Shaza Alsibai (Queen's University)
- Jacques Bélair (Université de Montréal)
- Tyler Cassidy (University of Leeds / University of British Columbia)
- Alexey Eremin (Saint Petersburg State University) [video]
- Nicola Guglielmi (Gran Sasso Science Institute) [video]
- Tony Humphries (McGill University)
- Jean-Philippe Lessard (McGill University)
- Hinke Osinga (University of Auckland)
- Stefan Ruschel (University of Nottingham)
- Francesca Scarabel (University of Leeds)
- Jan Sieber (University of Exeter)
- Rossana Vermiglio (Università degli Studi di Udine)
- Gail Wolkowicz (McMaster University)

Programme Schedule

All talks take place in Chernoff Hall Room 213. Coffee breaks are in Chernoff Hall Room 211. Lunch (June 10–12) is at Ban Righ Hall.

Wednesday, June 10

8:30–9:00	Coffee and welcome remarks
9:00–10:00	Morgan Craig
10:00–10:30	Coffee Break
10:30–11:00	Tyler Cassidy
11:00–11:30	Gail Wolkowicz
11:30–12:00	Shaza Alsibaa
12:00–13:30	Lunch
13:30–14:30	Bernd Krauskopf
14:30–15:00	Coffee Break
15:00–17:00	Early Career Session

Thursday, June 11

9:00–10:00	Sue Ann Campbell
10:00–10:30	Coffee Break
10:30–11:00	Jan Sieber
11:00–11:30	Francesca Scarabel
11:30–12:00	Jacques Bélair
12:00–13:30	Lunch
13:30–14:30	Renato Calleja
14:30–15:00	Coffee Break
15:00–15:30	Stefan Ruschel
15:30–16:00	Rossana Vermiglio
16:00–16:30	Hinke Osinga
16:30–17:00	Tony Humphries

Friday, June 12

9:00–10:00	Dimitri Breda
10:00–10:30	Coffee Break
10:30–11:00	Jean-Philippe Lessard
11:00–11:30	Nicola Guglielmi
11:30–12:00	Alexey Eremin
12:00–13:30	Lunch

Wednesday, June 10

Morgan Craig (Université de Montréal)

Mechanisms of immune regulation in response to viral infections and vaccines in healthy and immunocompromised hosts

The immune system is regulated by multiple nonlinear feedback loops, resulting in complex dynamical immune responses. Understanding the regulation of immunity is critical to identifying populations more at-risk of severe disease after infection and tailoring vaccination schedules to boost protection in vulnerable people. In this talk, I will discuss our work to unravel the mechanisms of immune regulation in response to viral infections and vaccines in healthy and immunocompromised hosts using mechanistic mathematical models. Owing to the complexity of the immune system, these models integrate delays to account for cellular and cytokine stimulation and inhibition. I will discuss how we use our models to generate virtual patient cohorts and run virtual clinical trial, ultimately identifying how multiple factors, including age and comorbidities, impact the immune response to pathogens and vaccines.

Tyler Cassidy (University of Leeds)

Efficient simulation and inference in delayed biological models

Delayed processes are ubiquitous throughout biology. Delay differential equations represent a natural modelling framework to capture these delayed dynamics. However, these delay differential equations can be difficult to simulate efficiently, which complicates parameter inference. Here, I'll discuss recent work that facilitates efficient model simulation and parameter inference without imposing artificial constraints on biological processes, and I'll illustrate the computational benefits of these approaches through a series of examples from across mathematical medicine.

Gail Wolkowicz (McMaster University)

A predator-prey model with delay in both the prey and predator growth terms

Previous research on a two-species Gause type predator-prey model with mass action interaction between the prey and the predator showed that the introduction of delay in the predator growth term can result in exotic dynamics in the form of stable periodic orbits with large or small amplitude, and a cascade of period doubling bifurcations leading to chaos. In this thesis, we formulate a predator-prey model with delay in the growth terms of both the predator and the prey. We assume that each population must survive the delay period, consistent with the decay terms in the model, in order to contribute to the growth of their respective populations. We use analytical methods to establish conditions on the effect of delay in the prey growth term on the existence and stability of equilibria, as well as the persistence of the prey population, with and without delay in the predator population growth term. In addition, we use numerical methods to show the effects of delay in the prey growth term on dynamics of the model with delay in the predator growth term. Short delays in the prey growth term can stabilize the interior equilibrium in regions displaying oscillations caused by delay in the predator growth term. Delay in the prey growth term can reduce the amplitude of periodic orbits, and trigger period-halving bifurcations in chaotic regions, ultimately stabilizing the interior equilibrium. Longer delays in the prey growth term can result in the extinction of the predator

population. Even longer delays in the prey growth term are shown to decrease the size of the prey population and lead to its extinction. This study extends previous findings on the stabilizing effect of delay in the prey growth term on oscillatory predator-prey systems to a new class of models.

Joint work with Konstantin Lenz-Gil and Christopher Greyson-Gaito

Shaza Alsibai (Queen’s University)

[Title to follow]

[Abstract to follow.]

Bernd Krauskopf (University of Auckland)

Resonances in DDE climate models: a conceptual view of El Niño

Feedback loops in climate systems arise from interactions between various subsystems, such as distinct bodies of water, the atmosphere, land and ice masses. Importantly, each feedback takes effect only after an inherent time delay. Such delayed feedback loops give rise to conceptual models in the form of DDEs. We consider here as a concrete example a conceptual DDE model for the El Niño Southern Oscillation phenomenon (ENSO). This DDE describes the interplay between a negative feedback loop and seasonal forcing. We show how El Niño events arise from ‘imperfect locking’ due to resonances between forcing and delay. We also discuss when and how state-dependence of the delay may play an important role for the observed resonance structure.

This is joint work with Andrew Keane (University College Cork), Claire Postlethwaite (University of Auckland) and Henk Dijkstra (Utrecht University).

Early Career Session

[Session titles and abstracts to be inserted]

Thursday, June 11

Sue Ann Campbell (University of Waterloo)

Time delays, Hopf bifurcation and synchronization

We consider networks of oscillator nodes with time delayed, global circulant coupling. We first study the existence of Hopf bifurcations induced by coupling time delay and then use symmetric Hopf bifurcation theory to determine how these bifurcations lead to different patterns of phase-locked oscillations. We apply the theory to a variety of systems inspired by biological neural networks to show how Hopf bifurcations can determine the synchronization state of the network. We show how the network structure can influence the existence of co-dimension two double Hopf bifurcations which in turn affects whether the nodes synchronize. We consider both discrete and distributed delays and determine how the type of distribution and size of the mean delay affect the results.

Jan Sieber (University of Exeter)

Neutral delay differential equations with state-dependent delays

We will show which results and numerical methods can still be applied when studying neutral delay differential equations where state-dependent delays occur in the neutral terms (so the delayed derivatives depend on state-dependent delays). In particular, the Hopf bifurcation theorem still holds, when one imposes a smallness condition on the delayed derivatives that is compatible with what one can expect for state-dependent delays.

This is joint work with Ahlam Alhadbani.

Francesca Scarabel (University of Leeds)

Investigating multistability and basins of attraction in models with time delay

In this talk I will present a recent approach to investigate multistability and the basins of attraction of dynamical systems with time delays. First, an approach to sample initial conditions and automatically detect the attractors of the system is proposed. Then, statistical dimensionality reduction techniques are used to extract important features of the initial conditions to interpret and visualise the basins of attractions of the system. The technique is illustrated using an infectious disease model with waning and boosting of immunity that admits multistability and other complex behaviours [1]. This work is part of an ongoing project with Evert Provoost (KU Leuven).

[1] Scarabel, F., Polner, M., Wylde, D., Barbarossa, M.V., and Röst, G. (2025) Bistability and complex bifurcation diagrams generated by waning and boosting of immunity, *Journal of Mathematical Biology*, 91:30. <https://doi.org/10.1007/s00285-025-02264-3>

Jacques Bélair (Université de Montréal)

Coupled delayed negative feedback loops in biological systems: oscillations and stability

Biological regulatory systems are often appropriately modeled as coupled systems of nonlinear delay differential equations. For example, the production of mammalian blood cells involves an inter-

twined network of physiological processes, with nonlinear, delayed feedback control mechanisms: erythrocytes (red blood cells) and thrombocytes (platelets) while each having their own main regulatory hormone, erythropoietin and thrombopoietin respectively, are also interacting, specially in pathological conditions. We consider the (highly) simplified model

$$\begin{aligned}x'(t) &= -\alpha x(t) + f(x(t - \tau), y(t\tau)) \\y'(t) &= -\beta y(t) + g(x(t - \tau), y(t - \tau))\end{aligned}$$

with f and g appropriate Hill functions for the coupled regulation of these two cell lines to study how the interaction of the control mechanisms may influence the dynamics. Equilibrium solutions are determined, their stability established and the nature of the oscillations when instability occurs are investigated. The linear analysis revolves around a transcendental characteristic equation of second order with two delays; a Centre manifold analysis at the change of stability of equilibria provides insight into possible dynamics. Possible extensions will also be discussed.

Renato Calleja (Universidad Nacional Autónoma de México)

Resonance phenomena in delay differential equations with two state-dependent delays

State-dependent delay differential equations arise naturally in applications where the delay is influenced by the current state of the system, leading to rich and often highly nontrivial dynamical behavior. In this talk, I will revisit joint work with Tony Humphries and Bernd Krauskopf on a scalar delay differential equation with two state-dependent delays, originally motivated by resonance phenomena and nonlinear oscillations. The equation exhibits intricate interactions between the delays and the oscillatory dynamics, including resonance tongues, phase-locking structures, and transitions between qualitatively different regimes. A central aspect of the analysis is the geometric organization of periodic solutions and bifurcations in parameter space, together with the numerical continuation techniques required to study such systems. Beyond the specific model, the work illustrates broader challenges and opportunities in the study of state-dependent delays, where analytical questions, numerical methods, and dynamical systems theory interact in subtle ways. I will also reflect on the influence that Tony Humphries has had on this area and on the collaboration that led to these results.

Stefan Ruschel (University of Nottingham)

Network dynamics driven by adaptive delays

We will present on frequency selection and network attractor formation in weakly nonlinear coupled systems by means of delay plasticity. Specifically, we consider adaptive axonal delays (AADs), motivated by activity-dependent myelination in the brain, which regulate signal propagation speeds and thus communication delays. We demonstrate frequency selection and network attractor formation in systems of delay-coupled phase oscillators with two prototypical choices of AADs on brain connectivity data and globally coupled ring networks.

Rossana Vermiglio (Università degli Studi di Udine)

[Title to follow]

[Abstract to follow.]

Hinke Osinga (University of Auckland)

Phase resetting in a system of coupled Van der Pol oscillators

Synchronisation properties of coupled nonlinear oscillators can be probed by studying the response to external perturbations: after relaxation back to the stable oscillation, there is generally a phase shift. Important information can be gained by studying such phase resets as a function of when the perturbation is applied during the oscillation. Phase resetting is a common tool in experiments on neurons. We developed a versatile and efficient numerical method for computing phase resetting curves for systems of any dimension; our approach is based on continuation of solutions to a multi-point boundary value problem. We present a case study of a prototypical example: two coupled 1:1 phase-locked Van der Pol oscillators. In contrast to single oscillators, this system has a phase space of dimension four. In particular, the basin of attraction of the stable synchronised oscillation has a complicated boundary, and we show how this affects the observed phase resetting in unexpected ways.

This is joint work with Kyoung Hyun Lee, Neil Broderick and Bernd Krauskopf.

Tony Humphries (McGill University)

A journey into delays

Threshold delays arise naturally in a wide variety of dynamical systems, including maturation and transport processes, and can be found in models of hematopoiesis, cell biology and immunology to name just a few examples. We'll finish this talk by discussing how to deal with them, and some of the issues that arise. But, with so many former students and collaborators and colleagues present, I'll spend most of this talk describing my own transport process from pure mathematics into numerical analysis and dynamical systems, then through advanced-retarded equations into delay equations and finally mathematical biology, and in particular some of the people that influenced me along the way.

Friday, June 12

Dimitri Breda (Università degli Studi di Udine)

On biological cascades and differential equations with advances

In natural and synthetic gene networks, Incoherent Feed Forward Loops (IFFLs) are known to generate pulses, accelerate response times, and filter input signals. While the behavior of a single IFFL has been extensively studied, far less is understood about circuits that include IFFL interconnections. A recent study on IFFLs interconnected in a cascade, a topology of potential interest for generating controllable temporal or spatial patterns in engineered or synthetic cells, shows how a pulsatile signal emerges in response to given inputs. The features of the pulse appear to converge to a coherent signal traveling along the cascade. A conjecture is discussed, suggesting that the asymptotic profile of the infinite IFFL cascade is characterized by a correlated differential equation with advance. This connection would help unravel the mechanism by which, along the cascade, the input to an IFFL is an advance version of its output.

This is a joint work with E. Franco, E. Nakamura and B. Whitley from UCLA and F. Blanchini from the University of Udine.

Jean-Philippe Lessard (McGill University)

Computer-assisted and constructive methods for periodic orbits in differential equations with large state-dependent delays

In this talk, we present a constructive, computer-assisted framework for proving the existence of periodic solutions in SDDEs with large state-dependent delays. The approach combines Fourier expansions with a Newton–Kantorovich argument in a low-regularity Banach space, together with rigorous treatment of composition operators via discrete Fourier analysis, contour integrals, and the discrete Poisson formula. We illustrate the method through three applications of increasing complexity: (1) a forced SDDE with prescribed period; (2) an autonomous SDDE where the period is part of the unknowns, following Calleja, Humphries, and Krauskopf (2018); and (3) an SDDE with a threshold nonlinearity, requiring unfolding parameters to handle the discontinuity and determine the period, based on Gedeon, Humphries, Mackey, Walther, and Wang (2025).

This is joint work with Jan Bouwe van den Berg (VU University Amsterdam), Maxime Breden (École Polytechnique), Matthieu Cadiot (École Polytechnique), and Kevin Church (CRM, Montréal).

Nicola Guglielmi (Gran Sasso Science Institute)

Solving time-fractional differential equations

A fractional differential equation is a generalization of an integer-order differential equation, where the (local) integer derivative operator is replaced by a (nonlocal) fractional derivative. With the help of the Riemann–Liouville fractional integral, such problems can be transformed into Volterra integro-differential equations. The main idea of our numerical approach is to approximate the fractional kernel by a sum of exponential functions, and then to transform the fractional integral (of convolution type) into a set of ordinary differential equations. Solving the resulting (stiff) system by a numerical time integrator leads to a fast and memory-efficient method. We also explain how

the code RADAU5 can be used to solve fractional differential equations. Numerical experiments illustrate the accuracy and efficiency of the proposed approach.

This is joint work with Ernst Hairer (Université de Genève).

Alexey Eremin (Saint Petersburg State University)

[Title to follow]

[Abstract to follow.]