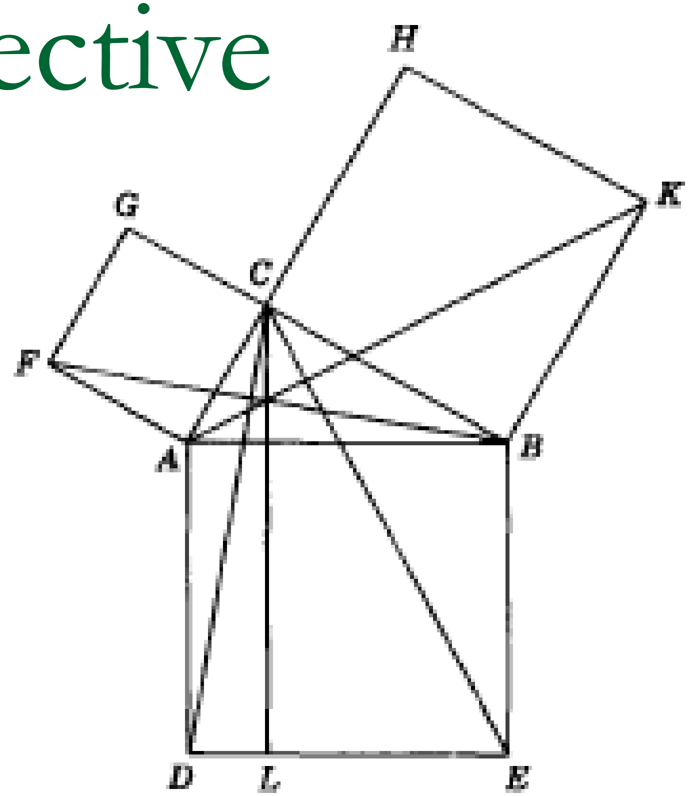
Math 381: Mathematics with a Historical Perspective

Introduction



Text and Grading

- Carl Boyer, A History of Mathematics, Wiley, Third Edition, 2011.
- Grading: 4 assignments worth 40% and
- 1 Final Examination worth 60%.
- Professor: M. Ram Murty (Office hours TBA)
- TA: Daniel Cloutier (Office hours TBA)
- Prerequisites: Undergraduate analysis and algebra

What is this course about?

- It is the study of the progression of mathematical ideas from ancient times to the modern period.
- The study has three components: mathematical, historical and literary.
- Part of our goal is to give a larger view of mathematics as an organic evolutionary discipline.
- Another component is to instruct the student in both literary and historical expression.

Outline

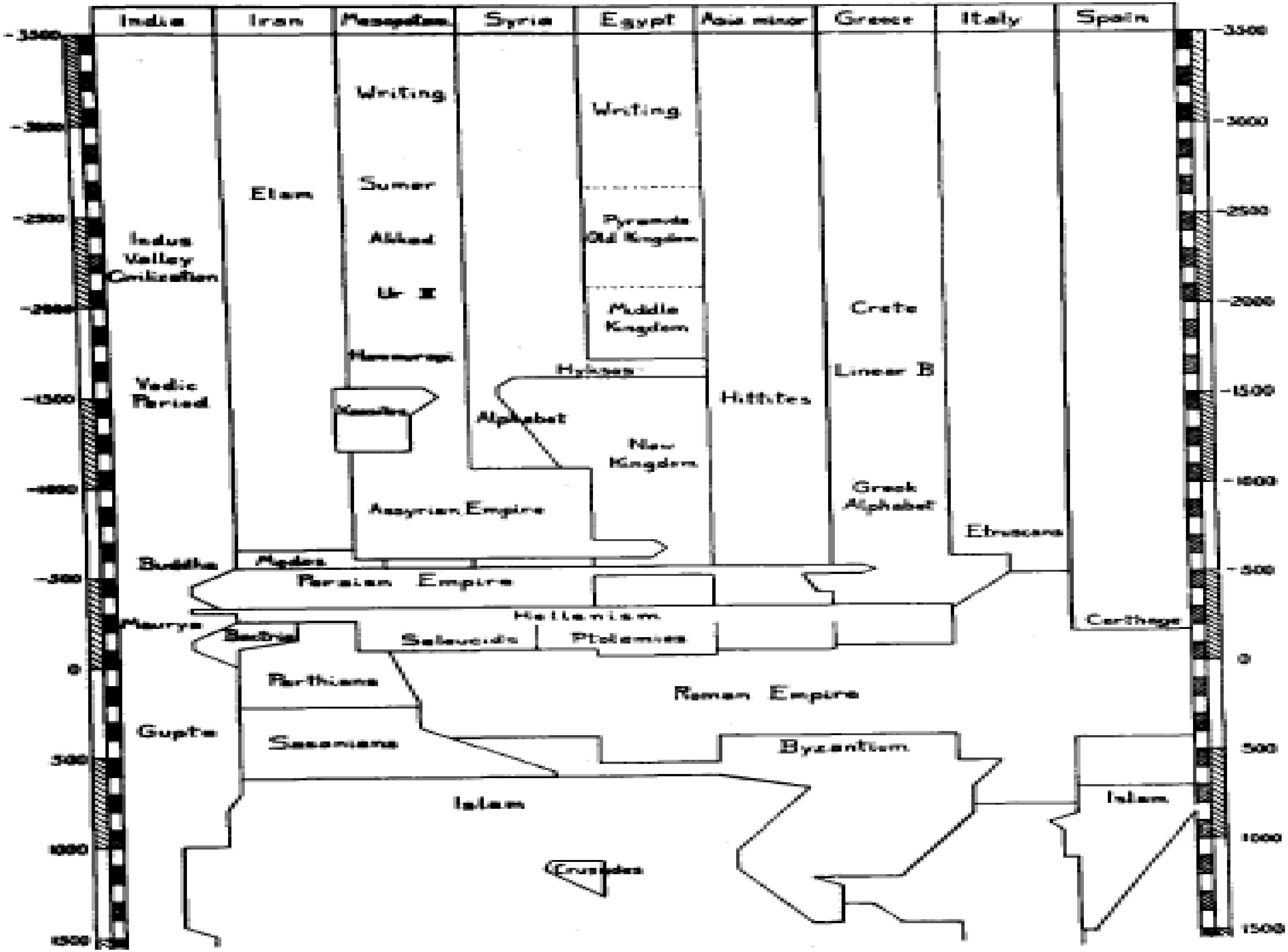
- 1. Origins: language, numbers and geometry.
- 2. Egypt: Ahmes papyrus, fractions, arithmetic.
- 3. Mesopatamia: Cuneiform records.
- 4. Pythagoras: ratio, proportion and mysticism.
- 5. Democritus: geometry and deduction.
- 6. Plato and Aristotle: Classical problems.
- 7. Euclid and Archimedes: number theory & geometry.
- 8. India and China: Sulvasutras, number systems and the birth of algebra.

Outline (continued)

- 9. The Arabic hegemony: algorithms & algebra.
- 10. Fibonacci: sequences and series.
- 11. The Renaissance: cubic and quartic equations.
- 12. Fermat and Descartes: beginnings of probability.
- 13. Newton and Leibniz: discovery of calculus
- 14. The Bernoullis: logarithms and probability
- 15. Euler: the development of number theory.
- 16. The French school: Laplace and Lagrange.
- 17. Gauss and Cauchy: Disquisitiones Arithmeticae.
- 18. Foundations: analysis and algebra

Outline (continued)

- 19. Poincare and Hilbert: modern perspectives.
- 20. Ramanujan and Hardy: East-West collaborations.
- 21. Lebesgue: theory of integration and probability.
- 22. Bourbaki: formalism of mathematics.
- 23. Logic and foundations: Godel's theorem.
- 24. The Future of mathematics: Fermat's last theorem.



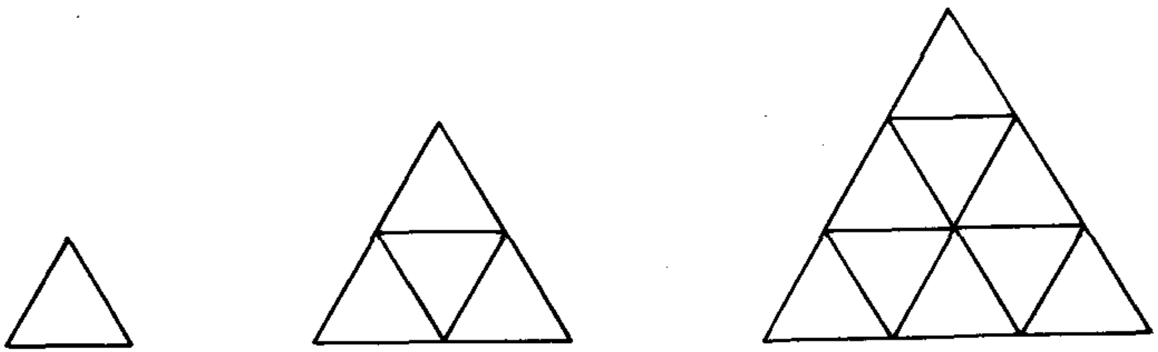
Chronological scheme representing the extent of some ancient and medieval civilizations. (Reproduced, with permission, from O. Neugebauer, *The Exact Sciences in Antiquity*.)

The concept of number

- The concept of a number appears in all ancient civilizations based on archeological evidence.
- Early number bases seem to be rooted in the fact that we have ten fingers and thus base 10 was natural.
- The representation of numbers using base 10 varied among different cultures. The positional notation in use today seems to have originated in India as early as 300 BCE.

Pottery of ancient Egypt

- New archeological evidence shows that the ancient Egyptians knew something about geometry.
- The construction of the pyramids needs knowledge of mathematics and its relation to spatial relations.



Ancient Babylon and Egypt

- The Behistun Cliff, discovered in the 1870's contained a trilingual account of the victory of Darius, in Persian, Eramitic and Babylonian.
- The Rosetta Stone (1799) also contained a trilingual account in Greek, Demotic and Hieroglyphics.
- We could now understand the meaning of many hieroglyphs found in the ancient pyramids.
- The ancient Egyptians had a crude positional notation for numbers: e.g. 12,345 was written as:

The Ahmes (Rhind) Papyrus and fractions

- In 1858, Henry Rhind found the Ahmes papyrus which seems to contain early ideas about fractions.
- For some reason, preference was given to fractions of the form 1/n. We call such fractions today as Egyptian fractions to honor this discovery.
- For example, for n odd, they would write:

$$\frac{2}{n} = \frac{1}{\frac{n+1}{2}} + \frac{1}{\frac{n(n+1)}{2}}$$

Arithmetic operations

- The basic operation in the Ahmes papyrus seems to be addition. Thus multiplication had to be reduced to addition through a technique of "doubling".
- For example, to calculate the product of 69 and 19, they would write $19=16+2+1$ so that $69(16+2+1)= 1104 + 138 + 69 = 1311$.
- We also see here a nascent form of binary representation of numbers.

Algebraic problems

- Rudimentary algebra problems are discussed in the papyrus. These are all linear equations of the $ax+b = c$.
- The unknown ‘x’ was called ‘aha’.
- The solution was arrived at by what can be called a clever ‘trial and error’ method.
- For example, problem 24 of the papyrus asks for the solution of $x + x/7 = 19$.
- They would guess the answer to be 7 but then the sum turns out to be 8.
- But $8(2+ 1/4 + 1/8) = 19$, so the correct answer is $7(2+1/4+1/8)$.

Geometric problems

- The ancient Egyptians had some idea about areas of squares and circles. They knew that the area of a circle of radius r is approximately $(19/6)r^2$.
- This is a poor approximation to π.
- The Moscow Papyrus, discovered in 1893, contains various problems about computation of volumes.
- It contains the correct formula for the volume of a frustum.

Volume of a frustum

- Suppose the base of the frustum is a square of side length a and the top is a square of side length b and the height is h.
- The volume is $h(a^2 + ab + b^2)/3$.
- Let's see why: we decompose the frustum as indicated: $b^2h + b(a-b)h + (a-b)^2h/3$