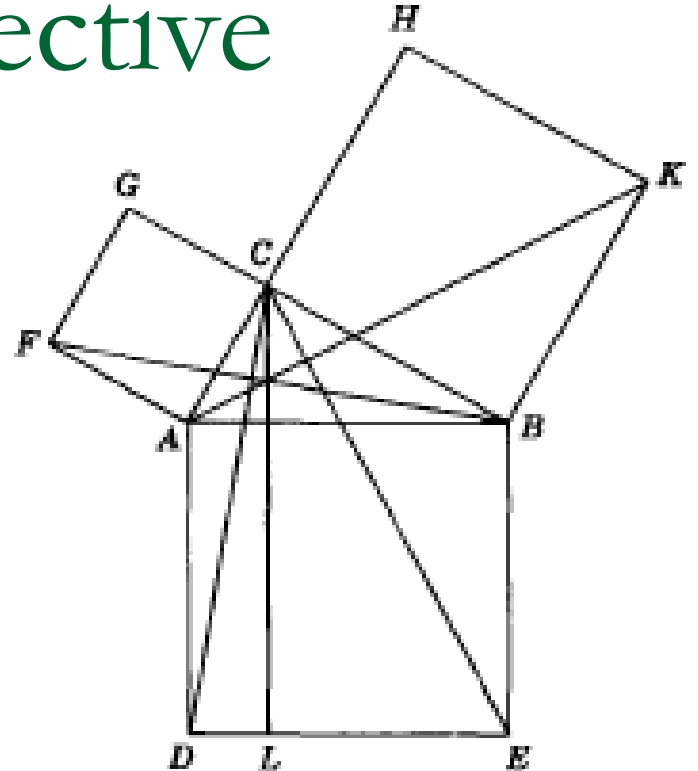


Math 381: Mathematics with a Historical Perspective

Introduction



Text and Grading

- Carl Boyer, *A History of Mathematics*, Wiley, Third Edition, 2011.
 - Grading: 4 assignments worth 40% and
 - 1 Final Examination worth 60%.
 - Professor: M. Ram Murty (Office hours TBA)
 - TA: Daniel Cloutier (Office hours TBA)
 - Prerequisites: Undergraduate analysis and algebra
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What is this course about?

- It is the study of the progression of mathematical ideas from ancient times to the modern period.
 - The study has three components: mathematical, historical and literary.
 - Part of our goal is to give a larger view of mathematics as an organic evolutionary discipline.
 - Another component is to instruct the student in both literary and historical expression.
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Outline

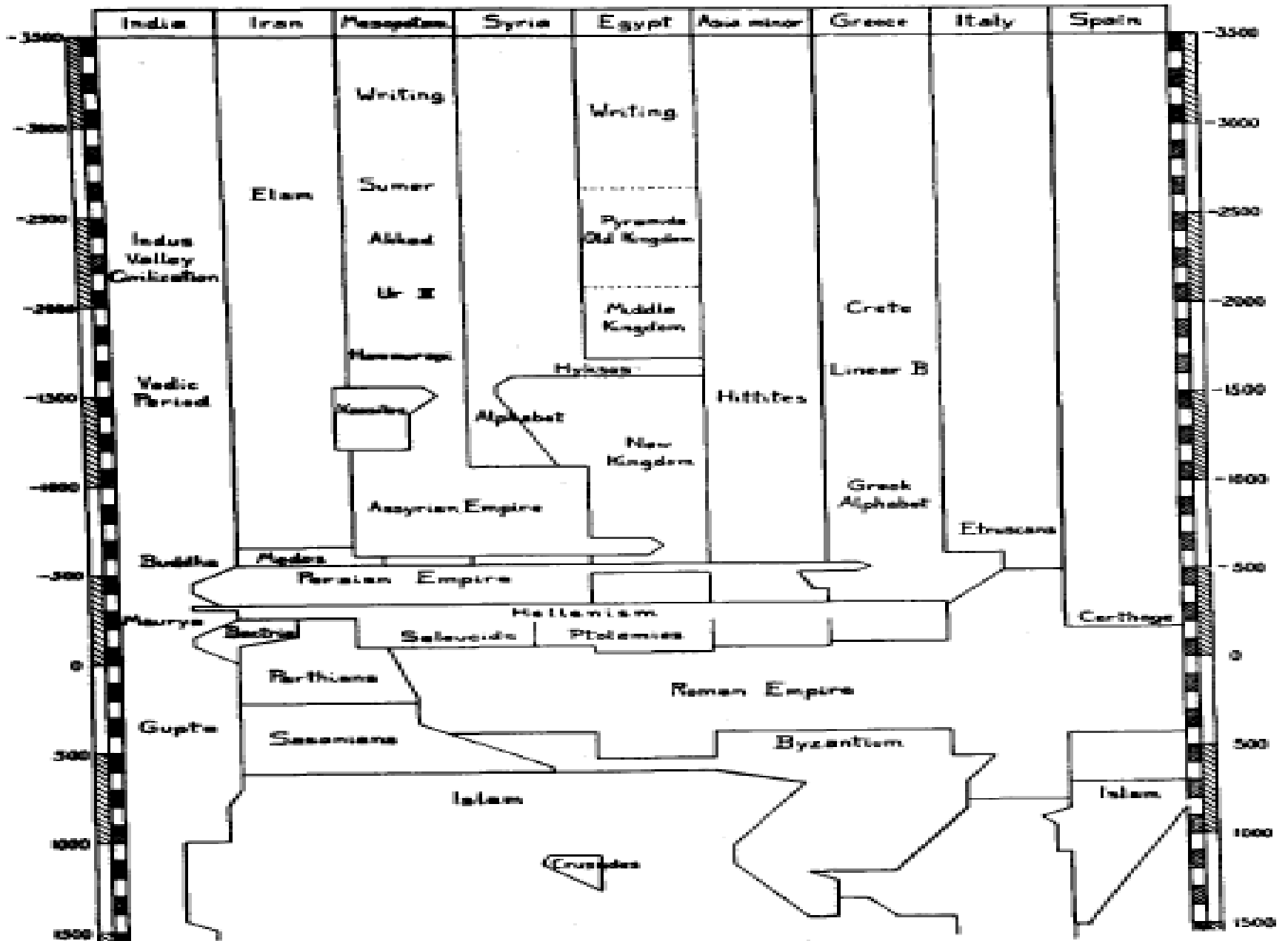
- 1. Origins: language, numbers and geometry.
 - 2. Egypt: Ahmes papyrus, fractions, arithmetic.
 - 3. Mesopotamia: Cuneiform records.
 - 4. Pythagoras: ratio, proportion and mysticism.
 - 5. Democritus: geometry and deduction.
 - 6. Plato and Aristotle: Classical problems.
 - 7. Euclid and Archimedes: number theory & geometry.
 - 8. India and China: Sulvasutras, number systems and the birth of algebra.
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Outline (continued)

- 9. The Arabic hegemony: algorithms & algebra.
- 10. Fibonacci: sequences and series.
- 11. The Renaissance: cubic and quartic equations.
- 12. Fermat and Descartes: beginnings of probability.
- 13. Newton and Leibniz: discovery of calculus
- 14. The Bernoullis: logarithms and probability
- 15. Euler: the development of number theory.
- 16. The French school: Laplace and Lagrange.
- 17. Gauss and Cauchy: *Disquisitiones Arithmeticae*.
- 18. Foundations: analysis and algebra

Outline (continued)

- 19. Poincare and Hilbert: modern perspectives.
 - 20. Ramanujan and Hardy: East-West collaborations.
 - 21. Lebesgue: theory of integration and probability.
 - 22. Bourbaki: formalism of mathematics.
 - 23. Logic and foundations: Godel's theorem.
 - 24. The Future of mathematics: Fermat's last theorem.
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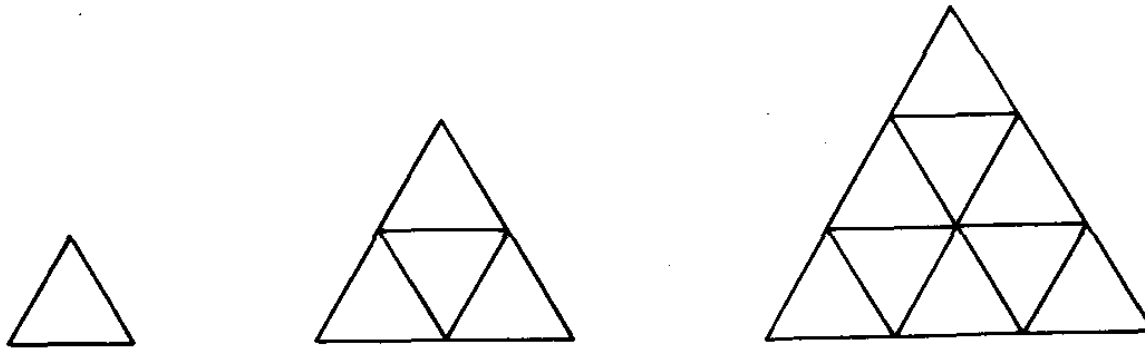
Chronological scheme representing the extent of some ancient and medieval civilizations. (Reproduced, with permission, from O. Neugebauer, *The Exact Sciences in Antiquity*.)

The concept of number

- The concept of a number appears in all ancient civilizations based on archeological evidence.
 - Early number bases seem to be rooted in the fact that we have ten fingers and thus base 10 was natural.
 - The representation of numbers using base 10 varied among different cultures. The positional notation in use today seems to have originated in India as early as 300 BCE.
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Pottery of ancient Egypt

- New archeological evidence shows that the ancient Egyptians knew something about geometry.
- The construction of the pyramids needs knowledge of mathematics and its relation to spatial relations.



Ancient Babylon and Egypt

- The Behistun Cliff, discovered in the 1870's contained a trilingual account of the victory of Darius, in Persian, Eramitic and Babylonian.
- The Rosetta Stone (1799) also contained a trilingual account in Greek, Demotic and Hieroglyphics.
- We could now understand the meaning of many hieroglyphs found in the ancient pyramids.
- The ancient Egyptians had a crude positional notation for numbers: e.g. 12,345 was written as:

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The Ahmes (Rhind) Papyrus and fractions

- In 1858, Henry Rhind found the Ahmes papyrus which seems to contain early ideas about fractions.
- For some reason, preference was given to fractions of the form $1/n$. We call such fractions today as Egyptian fractions to honor this discovery.
- For example, for n odd, they would write:

$$\frac{2}{n} = \frac{1}{\frac{n+1}{2}} + \frac{1}{\frac{n(n+1)}{2}}$$

Arithmetic operations

- The basic operation in the Ahmes papyrus seems to be addition. Thus multiplication had to be reduced to addition through a technique of “doubling”.
- For example, to calculate the product of 69 and 19, they would write $19=16+2+1$ so that $69(16+2+1)=1104 + 138 + 69 = 1311$.
- We also see here a nascent form of binary representation of numbers.

Algebraic problems

- Rudimentary algebra problems are discussed in the papyrus. These are all linear equations of the $ax+b = c$.
- The unknown 'x' was called 'aha'.
- The solution was arrived at by what can be called a clever 'trial and error' method.
- For example, problem 24 of the papyrus asks for the solution of $x + x/7 = 19$.
- They would guess the answer to be 7 but then the sum turns out to be 8.
- But $8(2 + 1/4 + 1/8) = 19$, so the correct answer is $7(2 + 1/4 + 1/8)$.

Geometric problems

- The ancient Egyptians had some idea about areas of squares and circles. They knew that the area of a circle of radius r is approximately $(19/6)r^2$.
 - This is a poor approximation to π .
 - The Moscow Papyrus, discovered in 1893, contains various problems about computation of volumes.
 - It contains the correct formula for the volume of a frustum.
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Volume of a frustum

- Suppose the base of the frustum is a square of side length a and the top is a square of side length b and the height is h .
- The volume is $h(a^2 + ab + b^2)/3$.
- Let's see why: we decompose the frustum as indicated: $b^2h + b(a-b)h + (a-b)^2h/3$

