20th century mathematics: A short survey

Hilbert

Hardy

Ramanujan

Selberg

Weil

Langlands

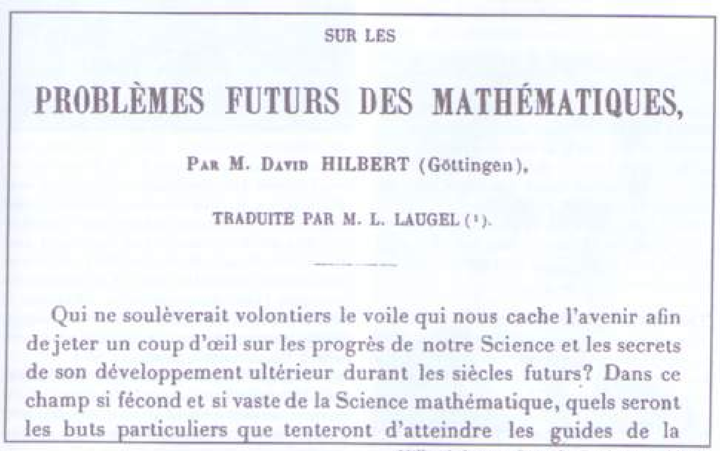
Hilbert's problems

- At the International Congress of Mathematics held in Paris in 1900, David Hilbert presented 23 unsolved problems which he felt the mathematical community should focus on and the solutions of which will lead to greater understanding of our world.
- The problems were definitely influential and 20th century can take some pride in that some of these problems are now solved and that we do have a better understanding of mathematics.
- A well-posed problem in mathematics serves to focus our energies in a coherent direction.

David Hilbert (1862-1943)

A look at some of Hilbert's problems

- The first few of his problems dealt with logic and foundations of set theory, a topic necessitated by the lack of mathematical rigor in some fields during the 19th century.
- It is not an exaggeration to say that each problem led to the development of a new branch of mathematics or stimulated the growth of existing branches in a fundamental way.
- The 7th problem led to the development of transcendental number theory
- The 8th problem is the Riemann hypothesis.
- The 9th problem led to the development of reciprocity laws.



SUR LES

PROBLÈMES FUTURS DES MATHÉMATIQUES,

PAR M. DAVID HILBERT (Göttingen),

TRADUITE PAR M. L. LAUGEL (¹).

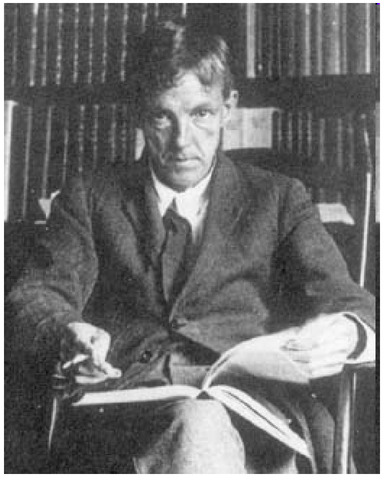
Qui ne soulèverait volontiers le voile qui nous cache l'avenir afin de jeter un coup d'œil sur les progrès de notre Science et les secrets de son développement ultérieur durant les siècles futurs? Dans ce champ si fécond et si vaste de la Science mathématique, quels seront les buts particuliers que tenteront d'atteindre les guides de la

Hilbert's lecture from the Paris Proceedings

The 10th problem led to the development of logic and diophantine set theory.
The 11th problem led to the arithmetic theory of quadratic forms.
The 12th problem led to class field theory.

The story of Hardy and Ramanujan

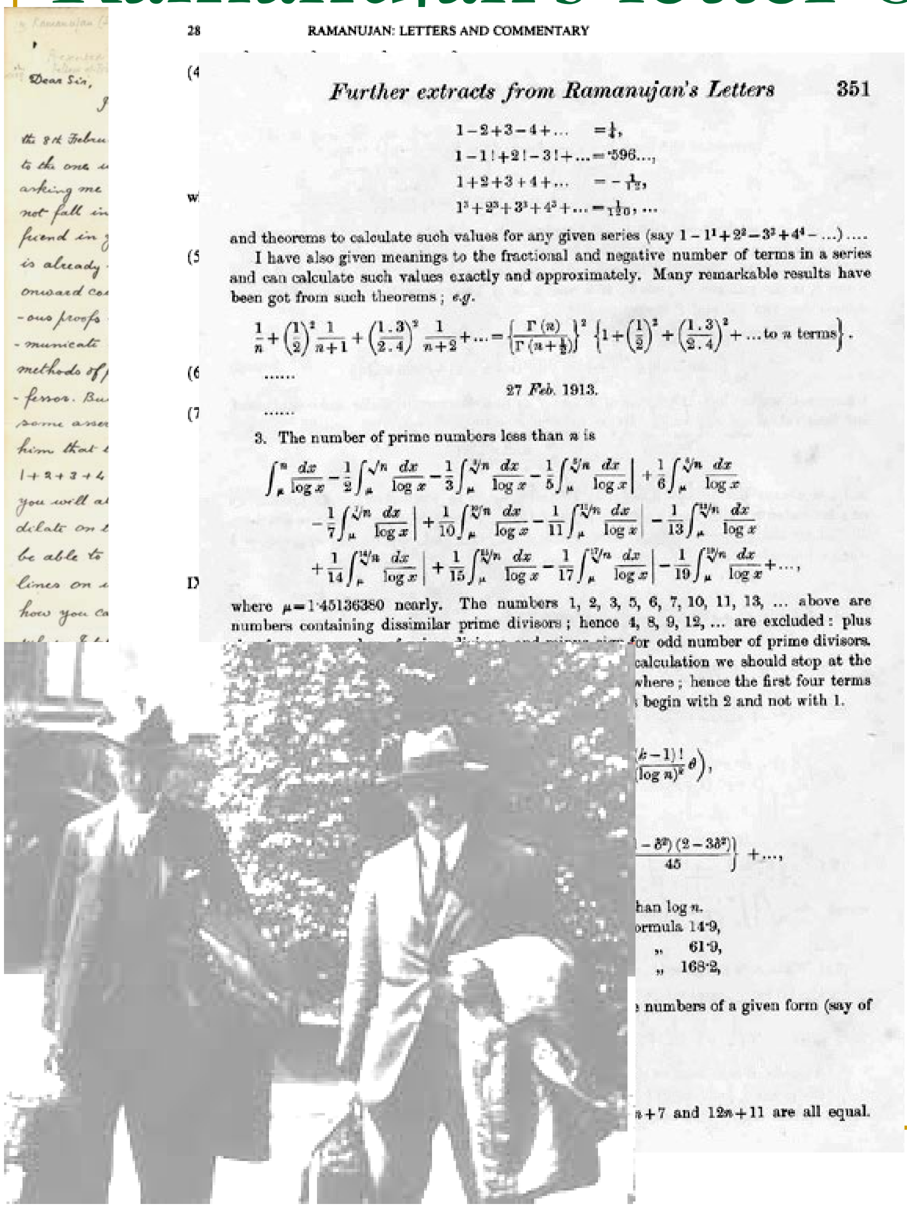
- Perhaps the most "romantic" of episodes in 20th century mathematics was the collaboration of the British mathematician G.H. Hardy and the Indian mathematician Srinivasa Ramanujan.
- Hardy was a well-known and distinguished mathematician at Trinity College of Cambridge University. Ramanujan was an unknown postal clerk and a self-taught mathematician who wrote to Hardy in his now famous 1913 letter that he had made some mathematical discoveries and would like to know if these results are new.
- When Hardy received the letter, he was flabbergasted and couldn't sleep that night. He spent all night poring over the letter and wondering how Ramanujan could have discovered them.
- The amazing thing about this is that it took place at a time when World War I was about to start and when India was under British rule.



G.H. Hardy(1877-1947)

Ramanujan (1887-1920)

Ramanujan's letter of 16 January 1913



28 RAMANUJAN: LETTERS AND COMMENTARY

Further extracts from Ramanujan's Letters 351

$$1-2+3-4+\ldots \quad = \tfrac{1}{4},$$
$$1-1!+2!-3!+\ldots = \cdot 596\ldots,$$
$$1+2+3+4+\ldots \quad = -\tfrac{1}{12},$$
$$1^3+2^3+3^3+4^3+\ldots = \tfrac{1}{120}, \ldots$$

and theorems to calculate such values for any given series (say $1-1^1+2^2-3^3+4^4-\ldots$) ....

I have also given meanings to the fractional and negative number of terms in a series and can calculate such values exactly and approximately. Many remarkable results have been got from such theorems; *e.g.*

$$\frac{1}{n}+\left(\frac{1}{2}\right)^2\frac{1}{n+1}+\left(\frac{1.3}{2.4}\right)^2\frac{1}{n+2}+\ldots=\left\{\frac{\Gamma(n)}{\Gamma(n+\frac{1}{2})}\right\}^2\left\{1+\left(\frac{1}{2}\right)^2+\left(\frac{1.3}{2.4}\right)^2+\ldots \text{to } n \text{ terms}\right\}.$$

......

27 *Feb.* 1913.

......

3. The number of prime numbers less than n is

$$\int_\mu^n \frac{dx}{\log x}-\frac{1}{2}\int_\mu^{\sqrt{n}}\frac{dx}{\log x}-\frac{1}{3}\int_\mu^{\sqrt[3]{n}}\frac{dx}{\log x}-\frac{1}{5}\int_\mu^{\sqrt[5]{n}}\frac{dx}{\log x}+\frac{1}{6}\int_\mu^{\sqrt[6]{n}}\frac{dx}{\log x}$$
$$-\frac{1}{7}\int_\mu^{\sqrt[7]{n}}\frac{dx}{\log x}+\frac{1}{10}\int_\mu^{\sqrt[10]{n}}\frac{dx}{\log x}-\frac{1}{11}\int_\mu^{\sqrt[11]{n}}\frac{dx}{\log x}-\frac{1}{13}\int_\mu^{\sqrt[13]{n}}\frac{dx}{\log x}$$
$$+\frac{1}{14}\int_\mu^{\sqrt[14]{n}}\frac{dx}{\log x}+\frac{1}{15}\int_\mu^{\sqrt[15]{n}}\frac{dx}{\log x}-\frac{1}{17}\int_\mu^{\sqrt[17]{n}}\frac{dx}{\log x}-\frac{1}{19}\int_\mu^{\sqrt[19]{n}}\frac{dx}{\log x}+\ldots,$$

where $\mu = 1{\cdot}45136380$ nearly. The numbers 1, 2, 3, 5, 6, 7, 10, 11, 13, ... above are numbers containing dissimilar prime divisors; hence 4, 8, 9, 12, ... are excluded: plus ... for odd number of prime divisors. ... calculation we should stop at the ... where; hence the first four terms ... begin with 2 and not with 1.

... han log n.
... ormula 14·9,
„ 61·9,
„ 168·2,

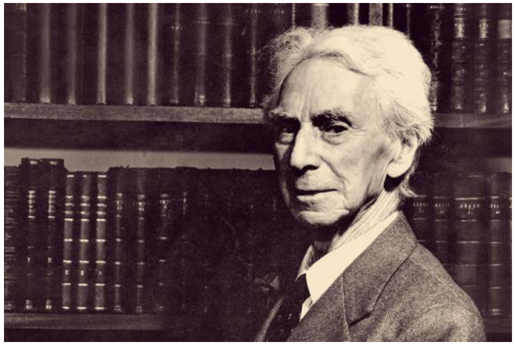
... numbers of a given form (say of

... $n+7$ and $12n+11$ are all equal.

- After losing sleep that night, Hardy invited his colleague, Littlewood to come over and have a look at Ramanujan's letter.
- After three hours, they concluded this was the work of a genius.

Hardy's reaction and reply to Ramanujan

- After a careful study of the 10-page letter which contained over a 120 theorems, Hardy concluded: "A single look at them is enough to show that they could only be written down by a mathematician of the highest class. They must be true because if they were not true, no one would have had the imagination to invent them."
- Bertrand Russell wrote that by the next day he "found Hardy and Littlewood in a state of wild excitement because they believe they have found a second Newton, a Hindu clerk in Madras making 20 pounds a year."



Hardy then arranged for Ramanujan to come over to England for research and collaboration.

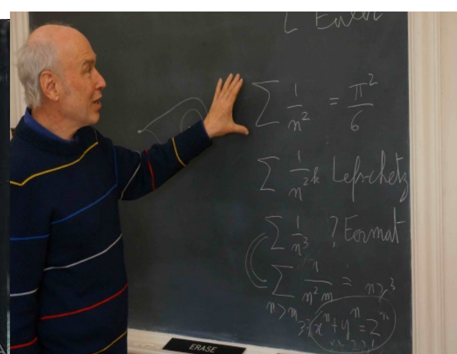
The Ramanujan conjectures

[redacted]

Ramanujan tau function

$$q\prod_{n\geq 1}(1-q^n)^{24} = \sum_{n\geq 1}\tau(n)q^n$$

$$\begin{aligned}\Delta(z) &= \frac{E_4(z)^3 - E_6(z)^2}{1728} \\ &= q - 24q^2 + 252q^3 - 1472q^4 + 4830q^5 - \\ &\quad 6048q^6 - 16744q^7 + 84480q^8 - 113643q^9 - 115920q^{10} + \\ &\quad 534612q^{11} - 370944q^{12} - 577738q^{13} + \\ &\quad 401856q^{14} + 1217160q^{15} + 987136q^{16} - \\ &\quad 6905934q^{17} + 2727432q^{18} + 10661420q^{19} - \\ &\quad 7109760q^{20} - 4219488q^{21} - 12830688q^{22} + \\ &\quad 18643272q^{23} + 21288960q^{24} - 25499225q^{25} + \\ &\quad 13865712q^{26} - 73279080q^{27} + 24647168q^{28} + \\ &\quad 128406630q^{29} - 29211840q^{30} + \ldots\end{aligned}$$



- In 1916, Ramanujan made the following conjectures:
- τ is multiplicative: $\tau(mn)=\tau(m)\tau(n)$ whenever m and n are coprime.
- τ satisfies a second order recurrence relation for prime powers.
- $|\tau(p)| < 2p^{11/2}$ for primes p.
- The first two conjectures were proved in 1917 by Mordell but he failed to see a general theory of modular forms nascent in Ramanujan's conjectures. This was discovered by Erich Hecke in 1936.

The last conjecture was proved by Deligne in 1974.

The Ramanujan zeta function

- Using his τ-function, Ramanujan constructed a new "zeta-like" function and conjectured that his zeta function has properties similar to the Riemann zeta function.
- Some of his predictions were proved later by Erich Hecke.
- So what is this analogy?

$$\sum_{n=1}^{\infty} \tau(n)/n^s$$

$$\prod_p \frac{1}{1-\tau(p)\,p^{-s}+p^{11-2s}}$$

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

$$\prod_p \frac{1}{1-1/p^s}$$

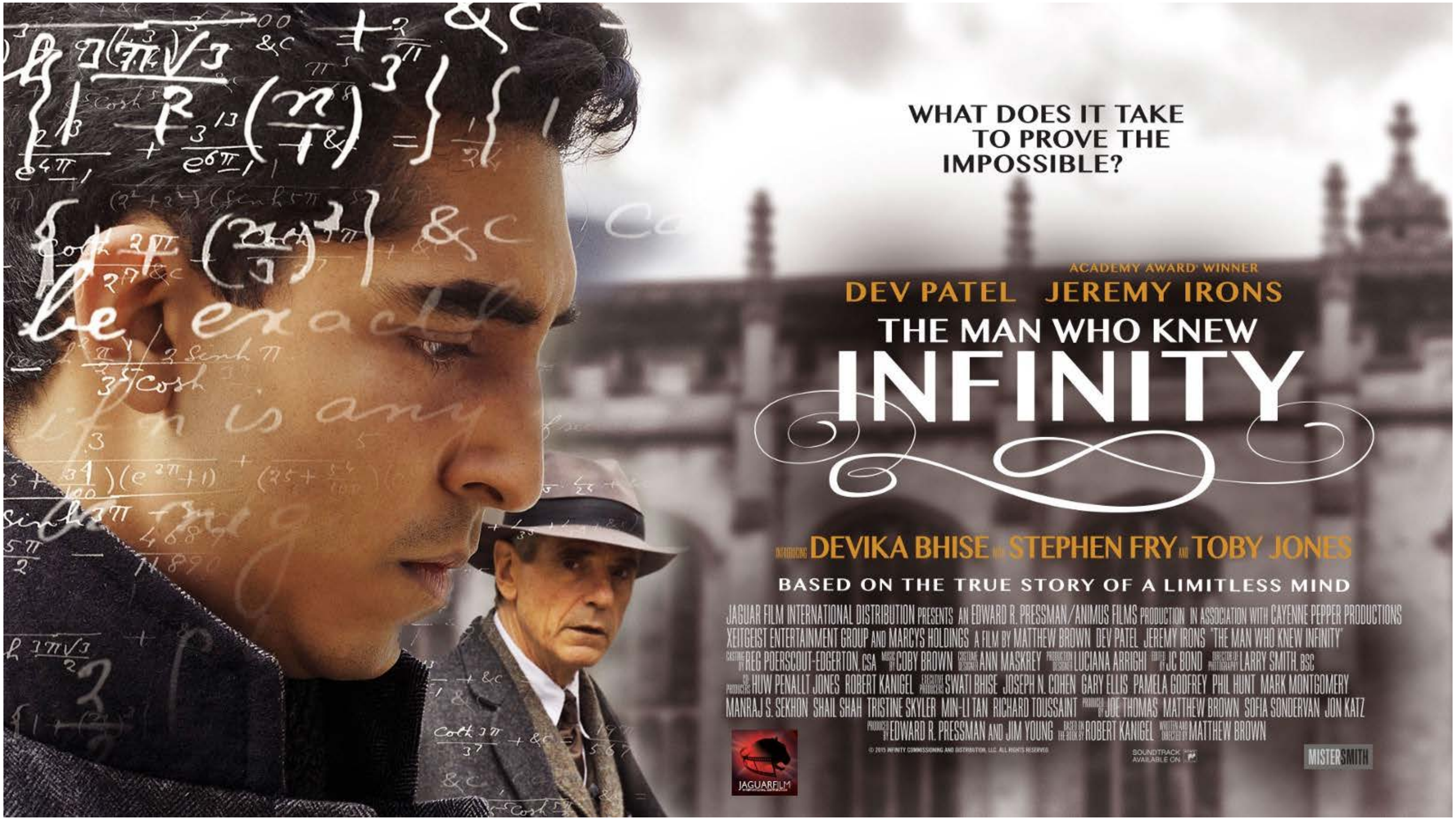
The famous taxicab story

$$
\begin{aligned}
(z) &= \frac{E_4(z)^3 - E_6(z)^2}{1728} \\
&= q - 24\,q^2 + 252\,q^3 - 1472\,q^4 + 4830\,q^5 - \\
&\quad 6048\,q^6 - 16744\,q^7 + 84480\,q^8 - 113643\,q^9 - 115920\,q^{10} + \\
&\quad 534612\,q^{11} - 370944\,q^{12} - 577738\,q^{13} + \\
&\quad 401856\,q^{14} + 1217160\,q^{15} + 987136\,q^{16} - \\
&\quad 6905934\,q^{17} + 2727432\,q^{18} + 10661420\,q^{19} - \\
&\quad 7109760\,q^{20} - 4219488\,q^{21} - 12830688\,q^{22} + \\
&\quad 18643272\,q^{23} + 21288960\,q^{24} - 25499225\,q^{25} + \\
&\quad 13865712\,q^{26} - 73279080\,q^{27} + 24647168\,q^{28} + \\
&\quad 128406630\,q^{29} - 29211840\,q^{30} + \ldots
\end{aligned}
$$

- There is an interesting story about this number connected to Ramanujan's life. When he was ill in a sanatorium, Hardy came to visit him in a taxi.
- On entering Ramanujan's room, Hardy remarked, "I just came in a taxi numbered 1729, which looks like a dull number. I hope it is not a bad omen."
- Ramanujan replied, "On the contrary, it is very interesting. It is the smallest number that can be written as the sum of two cubes in two different ways."

$$1729 = 1^3 + 12^3 = 9^3 + 10^3.$$

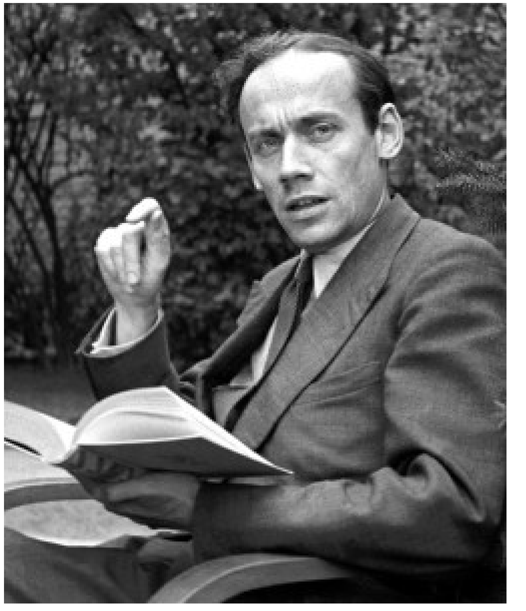
The man who knew infinity



Hilbert's problems, logic and foundations

- Several of Hilbert's problems deal with the foundations of mathematics, set theory and the nature of a proof.
- It is a remarkable achievement of the 20th century that these aspects were clarified.
- For instance, Cantor spent a good part of his life trying to prove the continuum hypothesis (Hilbert's first problem) and it was only in 1963 that Paul Cohen showed that it is independent of Zermelo-Frankel set theory, building on earlier work of Kurt Godel.
- Hilbert's 10th problem was also shown to be impossible in 1970 by Matiyasevich, building on earlier work of Davis and Robinson. This problem asked for a general algorithm for solving any given Diophantine equation.
- Godel's incompleteness theorem says that given any axiom system, there will be propositions that can be formulated in that system, that can neither be proved or disproved using that system.

The work of Emil Artin



- In his doctoral thesis, Emil Artin found an analogy between the ring of ordinary integers **Z** and the ring of polynomials (mod p), denoted $\mathbf{F}_p[x]$ and suggested that the study of this ring may be a way to understand the Riemann zeta function and perhaps the Riemann hypothesis.
- He was right! In his thesis, he constructed a new zeta function and verified (but could not prove) the analogue of the Riemann hypothesis in many cases.
- We can try to explain in simple terms the essence of Artin's conjecture as follows.
- Let N_p be the number of solutions of the congruence $y^2 = x^3 + ax + b \pmod{p}$ where the discriminant of the cubic is assumed to be non-zero mod p. Then $|N_p - p| \leq 2\sqrt{p}$.
- This was later proved in 1936 by Helmut Hasse.

An example

Let E be the curve $y^2 = x^3 + x + 1$ over $\mathbb{F}_5$.

x	x^2	$x^3 + x + 1$	y	Points
0	0	1	1, 4	(0, 1), (0, 4)
1	1	3	–	–
2	4	1	1, 4	(2, 1), (2, 4)
3	4	1	1, 4	(3, 1), (3, 4)
4	1	4	2, 3	(4, 2), (4, 3)

- There are a total of 8 solutions. Artin predicts that $|8 - 5| \leq 2\sqrt{5} = 4.47\ldots$ which is of course true.

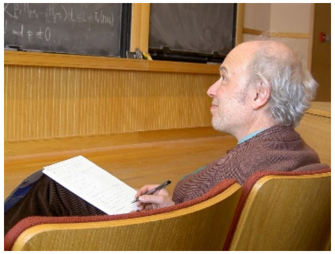
The Weil conjectures.

- Clearly, this is a special case of the more general problem of counting solutions to f(x,y)=0 (mod p) where f(x,y) is a polynomial in two variables.
- In 1948, Weil generalized Hasse's work to this context thereby creating a new branch of mathematics in algebraic geometry.
- Having done this, he mused about polynomials of several variables and made general conjectures, which he could not prove. This is contained in a famous paper of Weil written in 1949 and published in the Bulletin of the American Mathematical Society. Let us quote Weil himself on how he arrived at his conjectures.
- "In 1947, in Chicago, I felt bored and depressed, and, not knowing what to do, I started reading Gauss's two memoirs on biquadratic residues, which I have never read before. The first one deals with the number of solutions of $ax^4 - by^4 = 1$ over finite fields and the second one with $ax^3 - by^3 = 1$. Then I noticed similar principles can be applied to all equations of the form $ax^m + by^n + cz^r + \ldots = 0$ and that this implies the truth of the so-called Riemann hypothesis for diagonal equations."

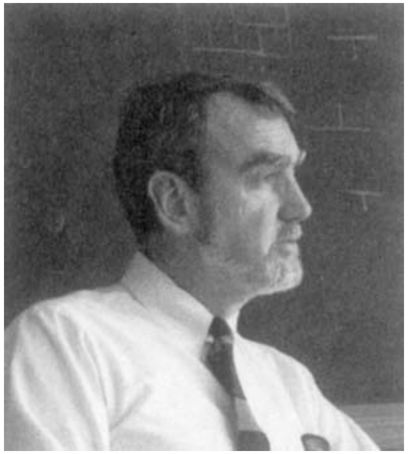
The work of Grothendieck & Deligne

- The Weil conjectures stimulated Grothendieck to re-think classical algebraic geometry and develop a new perspective.
- From this new perspective, the Weil conjectures would (should) be transparent.
- Weil made four conjectures regarding his new zeta functions, and Grothendieck could prove three of the four using his new theory.
- The fourth, called the analog of the Riemann hypothesis, which also had an intimate connection with Ramanujan's hypothesis about τ(p) was finally proved by Pierre Deligne in 1974.
- The key ingredients in Deligne's proof were a transfer of the method of Hadamard and de la Vallee Poussin in their proof of the prime number theorem along with a technique of Rankin and Selberg.

A. Grothendieck (1928-2014)



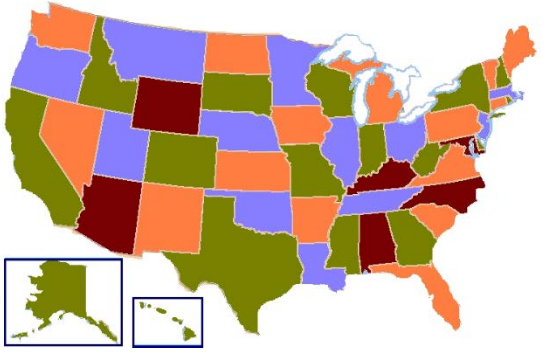
The Langlands program



- In the 1960's, Robert Langlands discovered that the Riemann zeta function, Ramanujan's zeta function are really special cases of a vast galaxy of L-functions attached to what are called automorphic representations.
- Though we have not solved the Riemann hypothesis, we see it as a special case of a large spectrum of related problems.

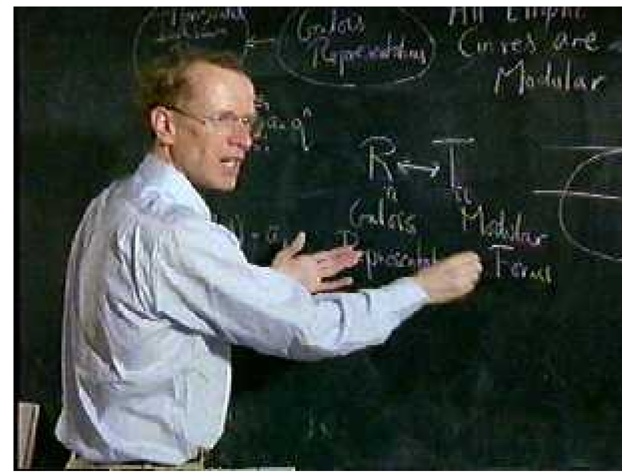
The four color theorem

- The four color conjecture was formulated in Francis Guthrie in 1852 and it says that any map can be properly colored using only four colors.
- A proper coloring of a map is such that no two adjacent regions get the same color.
- This turns out to be a difficult problem in graph theory although it looks like a coloring problem for kindergarten class!
- In 1890, Heawood showed five colors suffice but couldn't bring it down further.
- Finally, in 1976, Apell and Haken showed four colors suffice.
- Their proof used computers and so is not conceptual. In mathematics, we seek to discover concepts and not solve problems, problems being the occasion in which we seek to discover new concepts.



Taniyama's conjecture and Fermat's Last Theorem

- In an earlier lecture, we vaguely alluded to a connection between a conjecture of Taniyama regarding L-functions of elliptic curves and Fermat's Last Theorem.
- We also said that in 1993, Ken Ribet showed that Taniyama's conjecture implied Fermat's Last Theorem.
- The final decisive step was taken by Andrew Wiles combining his earlier work with Taylor, to prove Taniyama's conjecture.



Towards the twin prime problem

- One of Hilbert's problems alludes to the twin prime problem.
- This says there are infinitely many primes p such that p+2 is also prime.
- For example, (3,5), (5,7), (11,13), (17, 19), are examples of twin primes.

In 2013, Yitang Zhang showed there are infinitely many primes (p,q) such that |p-q| is bounded.

The story of how Yitang came to this discovery has an element of Ramanujan in it.

Counting from infinity

- Born in 1955 in Pinghu, China, Yitang Zhang could not go to school because of the cultural revolution. So he was largely self-taught.
- After the cultural revolution, he went to Peking University and obtained his BSc degree in 1982, at the age of 27.
- He always dreamed of doing number theory but his professors discouraged him in this direction and urged him to do algebra.
- He then went to Purdue University and completed a PhD in algebra in 1991.
- But he could not get an academic job, and so he did odd jobs, like a delivery boy or a waiter in a sandwich shop.
- Finally, in 1999, Kenneth Appel (of four color fame) hired him as an instructor at the University of New Hampshire.
- After teaching his classes, he would secretly work on the twin prime problem in the evenings until one day he made a breakthrough.
- On April 17, 2013, he quietly wrote up his paper and submitted it to the Annals of Mathematics, the top journal in mathematics.
- The paper was flawless and published the next month.
- There is a nice documentary called "Counting from infinity" that relates this story.

Chronology of mathematical developments

(Dates before −776 are approximations only)

−50,000	Evidence of counting
−25,000	Primitive geometrical designs
−4241	Hypothetical origin of Egyptian calendar
−3000	Hieroglyphic numerals in Egypt
−2773	Probable introduction of Egyptian calendar
−2400	Positional notation in Mesopotamia
−1850	Moscow (Golenishev) papyrus; cipherization

−5,000,000,000,000	Origin of the sun
−5,000,000,000	Origin of the earth
−600,000,000	Beginning of Paleozoic Age
−225,000,000	Beginning of Mesozoic Age
−60,000,000	Beginning of Cenozoic Age
−2,000,000	Origin of man
−50,000	Neanderthal Man
−25,000	Paleolithic art; Cro-Magnon Man
−10,000	Mesolithic agriculture
−5000	Neolithic civilizations
−4000	Use of metals
−3500	Use of potter's wheel; writing
−3000	Use of wheeled vehicles
−2800	Great Pyramid
−2400	Sumerian-Akkadian Empire
−1800	Code of Hammurabi
−1700	Hyksos domination of Egypt; Stonehenge in England
−1600	Kassite rule in Mesopotamia; New Kingdom in Egypt
−1400	Catastrophe in Crete

1100 BCE – 399 BCE

Chronological Table—*(contd.)*

–1100?	Chou-pei
–585	Thales of Miletus; deductive geometry (?)
–540	Pythagorean arithmetic and geometry (approx.) Rod numerals in China (approx.) Indian *Sulvasūtras* (approx.)
–450	Spherical earth of Parmenides (approx.)
–430	Death of Zeno; works of Democritus Astronomy of Philolaus (approx.) *Elements* of Hippocrates of Chios (approx.)
–428	Birth of Archytas; death of Anaxagoras
–427	Birth of Plato
–420	Trisectrix of Hippias (approx.) Incommensurables (approx.)
–369	Death of Theaetetus

–1350	Phonecian alphabet; use of iron; sundial; water clocks
–1200	Trojan War; Exodus from Egypt
–776	First Olympiad
–753	Traditional founding of Rome
–743	Era of Nabonassar
–740	Works of Homer and Hesoid (approx.)
–586	Babylonian Captivity
–538	Persians took Babylon
–480	Battle of Thermopylae
–477	Formation of Delian League
–461	Beginning of Age of Pericles
–430	Hippocrates of Cos (approx.) Atomic doctrine (approx.)
–429	Death of Pericles; plague at Athens
–404	End of Peloponnesian War
–399	Death of Socrates; *Anabasis* of Xenophon

360 BCE – 121 BCE

– 360 Eudoxus on proportion and exhaustion (approx.)
– 350 Menaechmus on conic sections (approx.)
Dinostratus on quadratrix (approx.)
– 335 Eudemus: *History of Geometry* (approx.)
– 330 Autolycus: *On the Moving Sphere* (approx.)
– 320 Aristaeus: *Conics* (approx.)
– 300 Euclid's *Elements* (approx.)
– 260 Aristarchus' heliocentric astronomy (approx.)
– 230 Seive of Eratosthenes (approx.)
– 225 *Conics* of Apollonius (approx.)
– 212 Death of Archimedes
– 180 Cissoid of Diocles (approx.)
Conchoid of Nicomedes (approx.)
Hypsicles and 360° circle (approx.)
– 150 Spires of Perseus (approx.)
– 140 Trigonometry of Hipparchus (approx.)

– 347 Death of Plato
– 332 Alexandria founded
– 323 Death of Alexander
– 322 Deaths of Aristotle and Demosthenes
– 311 Beginning of Seleucid Era in Mesopotamia
– 306 Ptolemy I (Soter) of Egypt
– 283 Pharos at Alexandria
– 264 First Punic War opened
– 232 Death of Asoka, the "Buddhist Constantine"
– 210 Great Chinese Wall begun
– 166 Revolt of Judas Maccabaeus
– 146 Destruction of Carthage and Corinth
– 121 Gaius Gracchus killed

60 BCE – 532 CE

Chronological Table—*(contd.)*

−60	Geminus on parallel postulate (approx.)	−75	Cicero restored tomb of Archimedes
		−60	Lucretius: *De rerum natura*
		−44	Death of Julius Caesar
+75	Works of Heron of Alexandria (approx.)	+79	Death of Pliny the Elder at Vesuvius
100	Nicomachus: *Arithmetica* (approx.) Menelaus: *Spherics* (approx.)		
		116	Trajan extends Roman Empire
		122	Hadrian's Wall in Britain begun
125	Theon of Smyrna and Platonic mathematics		
150	Ptolemy: *The Almagest* (approx.)	180	Death of Marcus Aurelius
250	Diophantus: *Arithmetica* (approx.?)	286	Division of Empire by Diocletian
320	Pappus: *Mathematical Collections* (approx.)	324	Founding of Constantinople
		378	Battle of Adrianople
390	Theon of Alexandria (fl.)		
415	Death of Hypatia	455	Vandals sack Rome
470	Tsu Ch'ung-chi's value of π (approx.)	476	Traditional "fall" of Rome
476	Birth of Aryabhata		
485	Death of Proclus	496	Clovis adopted Christianity
520	Anthemius of Tralles and Isidore of Miletus		
524	Death of Boethius	526	Death of Theodoric
529	Closing of the schools at Athens	529	Founding of the monastery at Monte Cassino
		532	Building of Hagia Sophia by Justinian

560-1170

560 Eutocius' commentaries on Archimedes (approx.)

628 Brahma-sphuta-siddhânta

662 Bishop Sebokht mentioned Hindu numerals

735 Death of the Venerable Bede
775 Hindu works translated into Arabic

830 Al-Khowarizmi: *Algebra* (approx.)
901 Death of Thabit ibn-Qurra

998 Death of abu'l-Wefa

1037 Death of Avicenna
1039 Death of Alhazen
1048 Death of al-Biruni

1114 Birth of Bhaskara
1123 Death of Omar Khayyam
1142 Adelard of Bath translated Euclid

1202 Fibonacci: *Liber abaci*

590 Gregory the Great elected pope
622 Hejira of Mohammed

641 Library at Alexandria burned

732 Battle of Tours

814 Death of Charlemagne

910 Benedictine abbey at Cluny
987 Accession of Hugh Capet

999 Gerbert became Pope Sylvester II
1028 School of Chartres

1066 Battle of Hastings
1096 First Crusade
1100 Henry I of England crowned

1170 Murder of Thomas à Becket

1260-1485

Chronological Table—*(contd.)*

1260 Campanus' trisection (approx.)
Jordanus Nemorarius: *Arithmetica* (approx.)

1270 Wm. of Moerbeke translated Archimedes (approx.)

1274 Death of Nasir Eddin

1303 Chu Shi-kié and the Pascal triangle
1328 Bradwardine: *Liber de proportionibus*
1336 Death of Richard of Wallingford

1360 Oresme's latitude of forms (approx.)

1436 Death of al-Kashi

1464 Death of Nicholas of Cusa
1472 Peurbach: *New Theory of the Planets*

1476 Death of Regiomontanus
1482 First printed Euclid

1484 Chuquet: *Triparty*

1204 Crusaders sack Constantinople
Death of Maimonides
1215 Magna Carta

1265 "First" parliament in England

1271 Travels of Marco Polo; mechanical clocks (approx.)

1286 Invention of eyeglasses (approx.)

1348 The Black Death

1364 Death of Petrarch
1431 Joan of Arc burned

1440 Invention of printing
1453 Fall of Constantinople

1473 Sistine Chapel

1483 Murder of the princes in the Tower

1485 Henry VII, first Tudor

1489-1609

1489 Use of + and − by Widmann
1492 Use of decimal point by Pellos
1494 Pacioli: *Summa*

1525 Rudolff: *Coss*
1526 Death of Scipione dal Ferro
1527 Apian published the Pascal triangle

1543 Tartaglia published Moerbeke's *Archimedes*
Copernicus: *De revolutionibus*
1544 Stifel: *Arithmetica integra*
1545 Cardan: *Ars magna*

1557 Recorde: *Whetstone of Witte*

1564 Birth of Galileo
1572 Bombelli: *Algebra*
1579 Viète: *Canon mathematicus*

1585 Stevin: *La disme*
Harriot's report on "Virginia"

1595 Pitiscus: *Trigonometria*

1603 Death of Viète
1609 Kepler: *Astronomia nova*
1614 Napier's logarithms

1492 Discovery of America by Columbus

1498 Execution of Savonarola
1517 Protestant Reformation
1520 Field of the Cloth of Gold

1534 Act of Supremacy
1543 Vesalius: *De fabrica*
Ramus: *Reproof of Aristotle*

1553 Servetus burned at Geneva

1558 Accession of Elizabeth I
1564 Birth of Shakespeare; deaths of Vesalius and Michelangelo
1572 Massacre of St. Bartholomew

1584 Assassination of William of Orange

1588 Drake defeated the Spanish Armada

1598 Edict of Nantes
1603 Deaths of Wm. Gilbert and Elizabeth I
1609 Galileo's telescope

1620-1666

Chronological Table—*(contd.)*

1620 Bürgi's logarithms

1629 Fermat's method of maxima and minima

1631 Harriot: *Artis analyticae praxis*

Oughtred: *Clavis mathematicae*

1635 Cavalieri: *Geometria indivisibilibus*

1637 Descartes: *Discours de la méthode*

1639 Desargues: *Bruillon projet*

1640 *Essay pour les coniques* of Pascal

1642 Birth of Newton; death of Galileo

1647 Deaths of Cavalieri and Torricelli

1655 Wallis: *Arithmetica infinitorum*

1657 Neil rectified his parabola

1658 Huygens' cycloidal pendulum clock

1667 Gregory: *Geometriae pars universalis*

1668 Mercator: *Logarithmotechnia*

1670 Barrow: *Lectiones geometriae*

1672 Assassination of De Witt

1616 Deaths of Shakespeare and Cervantes

1620 Landing of the Pilgrims

1626 Deaths of Francis Bacon and Willebrord Snell

1628 Harvey: *De motu cordis et sanguinis*

1636 Harvard College founded

1643 Accession of Louis XIV

1644 Torricelli's barometer

1649 Charles I beheaded

1651 Hobbes: *Leviathan*

Von Guericke's air pump

1660 The Restoration

1662 Royal Society founded

1666 Académie des Sciences founded

1678-1750

1678	Ceva's theorem		
		1679	Writ of Habeas Corpus
		1682	*Acta eruditorum* founded
		1683	Seige of Vienna
1684	Leibniz' first paper on the calculus		
		1685	Revocation of the Edict of Nantes
1687	Newton: *Principia*		
		1689	The Glorious Revolution
1690	Rolle: *Traité d'algèbre*		
1696	Brachistochrone (the Bernoullis) L'Hospital's rule		
		1699	Death of Racine
		1702	Opening of Queen Anne's War
1706	Use of π by William Jones		
		1711	Birth of Hume
1715	Taylor: *Methodus incrementorum*		
1718	De Moivre: *Doctrine of Chances*	1718	Fahrenheit's thermometer
1722	Cotes: *Harmonia mensurarum*		
1730	Stirling's formula	1730	Rèaumur's thermometer
1731	Clairaut on skew curves		
1733	Saccheri: *Euclid Vindicated*		
1734	Berkeley: *The Analyst*		
		1737	Linnaeus: *Systema naturae*
		1738	Daniel Bernoulli: *Hydrodynamica*
		1740	Accession of Frederick the Great
1742	Maclaurin: *Treatise of Fluxions*	1742	Centigrade thermometer
1743	D'Alembert: *Traité de dynamique*		
1748	Euler: *Introductio*; Agnesi: *Istituzioni*		
		1749	Volume I of Buffon's *Histoire naturelle*

1750-1815

1750 Cramer's rule; Fagnano's ellipse

1759 *Die freye Perspektive* of Lambert

1770 Hyperbolic trigonometry

1777 Buffon's needle problem
1779 Bézout on elimination

1788 Lagrange: *Mécanique analytique*

1794 Legendre: *Eléments de géométrie*
1795 Monge: *Feuilles d'analyse*
1796 Laplace: *Système du monde*
1797 Lagrange: *Fonctions analytiques*
Mascheroni: *Geometria del compasso*
Wessel: *Essay on . . . direction*
Carnot: *Métaphysique du calcul*

1801 Gauss: *Disquisitiones arithmeticae*
1803 Carnot: *Géométrie de position*

1810 Volume I of Gergonne's *Annales*

1751 Volume I of Diderot's *Encyclopédie*
1752 Franklin's kite

1767 Watt's improved steam engine

1774 Discovery of Oxygen (Priestley, Scheele, Lavoisier)
1776 American Declaration of Independence

1781 Discovery of Uranus by Herschel
1783 Composition of water (Cavendish, Lavoisier)

1789 French Revolution
1794 Lavoisier guillotined
1795 École Polytechnique; École Normale
1796 Vaccination (Jenner)

1799 Metric system
1800 Volta's battery
1801 Ceres discovered
1803 Dalton's atomic theory
1804 Napoleon crowned emperor

1814 Fraunhofer lines

1815-1850

1815 "The Analytical Society" at Cambridge
1817 Bolzano: *Rein analytischer Beweis*

1822 Poncelet: *Traité*; Fourier series; Feuerbach's theorem
1826 Crelle's *Journal* founded
Principle of Duality (Poncelet, Plücker, Gergonne)
Elliptic functions (Abel, Gauss, Jacobi)
1827 Homogeneous coordinates (Möbius, Plücker, Feuerbach)
Cauchy: *Calculus of Residues*
1828 Green: *Electricity and Magnetism*
1829 Lobachevskian geometry
Death of Abel at age 26
1830 Peacock: *Algebra*

1832 Bolyai: *Absolute Science of Space*
Death of Galois at age 20
1834 Steiner became professor at Berlin
1836 Liouville's *Journal* founded
1837 *Cambridge and Dublin Mathematical Journal*

1843 Hamilton's quaternions
1844 Grassmann: *Ausdehnungslehre*

1847 Von Staudt: *Geometrie der Lage*

1852 Chasles: *Traité de géométrie supérieure*

1815 Battle of Waterloo
1817 Optical transverse vibrations (Young and Fresnel)
1820 Oersted discovered electromagnetism

1826 Ampère's work in electrodynamics

1827 Ohm's law

1828 Synthesis of urea by Wöhler
1829 Death of Thomas Young

1830 Lyell: *Principles of Geology*
Comte: *Cours de philosophie positive*
1831 Faraday's electromagnetic induction
1832 Babbage's analytical engine

1836 First telegraph

1842 Conservation of energy (Mayer and Joule)

1846 Discovery of Neptune (Adams and Leverrier)
Use of anesthesia

1848 Marx: *Communist Manifesto*
1850 Dickens: *David Copperfield*

1850-1900

1854 Riemann's Habilitationschrift
Boole: *Laws of Thought*
1855 Dirichlet succeeded Gauss at Göttingen

1863 Cayley appointed at Cambridge
1864 Weierstrass appointed at Berlin

1872 Dedekind: *Stetigkeit und irrationale Zahlen*
Heine: *Elemente*
Méray: *Nouveau preçis*
Klein's Erlanger Programm
1873 Hermite proved e transcendental
1874 Cantor's *Mengenlehre*

1877 Sylvester appointed at Johns Hopkins
1881 Gibbs: *Vector Analysis*
1882 Lindemann proved π transcendental
1884 Frege: *Grundlagen der Arithmetik*

1888 Beginnings of American Mathematical Society
1889 Peano's axioms
1895 Poincaré: *Analysis situs*
1896 Prime number theorem proved (Hadamard and De la Vallée-Poussin)

1858 Atlantic cable
1859 Darwin: *Origin of Species*
Chemical spectroscopy (Bunsen and Kirchhoff)

1868 Cro-Magnon caves discovered
1869 Opening of Suez Canal
Mendeleef's periodic table

1873 Maxwell: *Electricity and Magnetism*

1876 Bell's telephone

1887 Discovery of herzian waves
1888 Pasteur Institute founded

1895 Discovery of X-rays (Roentgen)
1896 Discovery of radioactivity (Becquerel)

20th century

1899 Hilbert: *Grundlagen der Geometrie*
1900 Hilbert's problems
Volume I of Russell and Whitehead: *Principia*

1903 Lebesgue integration

1906 Functional calculus (Fréchet)
1907 Brouwer and intuitionism
1914 Hausdorff: *Grundzüge der Mengenlehre*

1916 Einstein's general theory of relativity
1917 Hardy and Ramanujan on theory of numbers
1923 Banach spaces

1930 Weyl succeeded Hilbert at Göttingen
1931 Gödel's theorem
1933 Weyl resigned at Göttingen
1934 Gelfond's theorem
1939 Volume I of Bourbaki: *Eléments*

1955 Homological algebra (Cartan and Eilenberg)
1963 Paul J. Cohen on continuum hypothesis

1966 15th International Congress of Mathematicians (Moscow)

1897 Discovery of electron (J. J. Thomson)
1898 Discovery of radium (Marie Curie)

1900 Freud: *Die Traumdeutung*

1901 Planck's quantum theory
1903 First powered air flight
1905 Special relativity (Einstein)

1914 Assassination of Austrian Archduke
1915 Panama Canal opened

1917 Russian Revolution

1927 Lindbergh flew the Atlantic
1928 Fleming discovered penicillin

1933 Hitler became chancellor

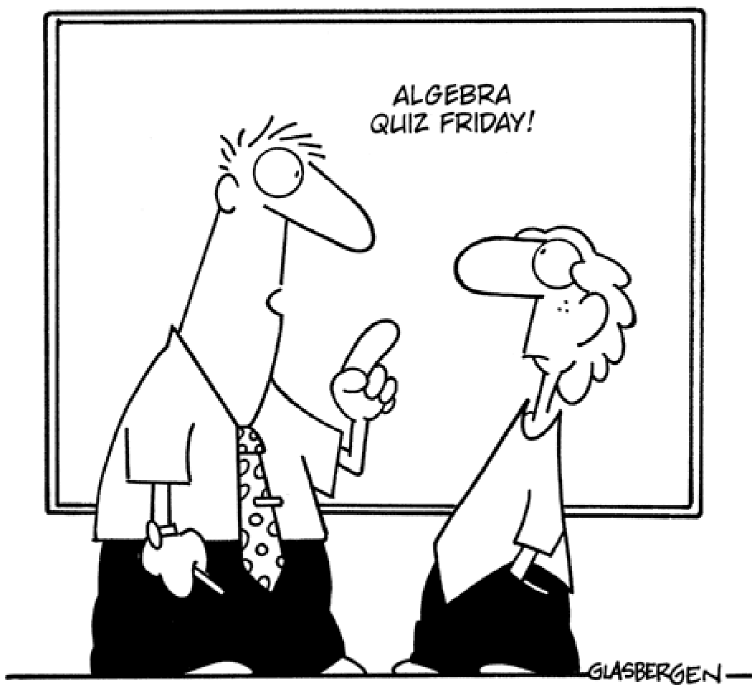
1941 Pearl Harbor
1945 Bombing of Hiroshima
1946 First meeting of U.N.

1963 Assassination of President Kennedy
1965 Death of Sir Winston Churchill

Later developments

- 1928 Waring's problem by Hardy and Littlewood.
- 1936 Godel's incompleteness theorem.
- 1963 Paul Cohen and the continuum hypothesis.
- 1970 Solution of Hilbert's tenth problem by Matiyasevich.
- 1974 Weil conjectures proved by Deligne.
- 1976 Four color theorem using computers.
- 1996 Fermat's Last Theorem by Andrew Wiles.
- 2013 Yitang Zhang proves bounded gaps between primes.

The importance of mathematics



"It's important to learn math because someday you might accidentally buy a phone without a calculator."