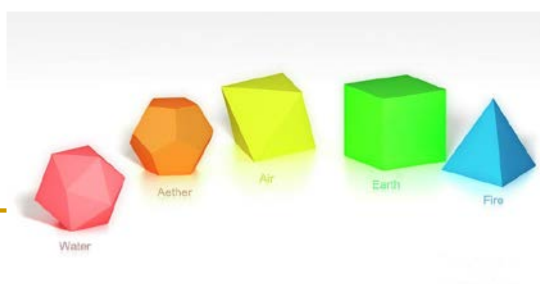
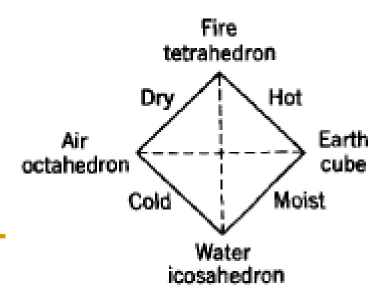
Plato and Aristotle

Plato’s Academy in Athens(400 BCE)

- Although Plato was more a philosopher rather than a mathematician, it is clear that he had a high regard for mathematics and was seen as a “maker of mathematicians.”
- At the entrance of his academy was written, “Let no one ignorant of geometry enter here.”
- The five Platonic solids were mystically associated with the four fundamental elements of earth, water, fire and air.





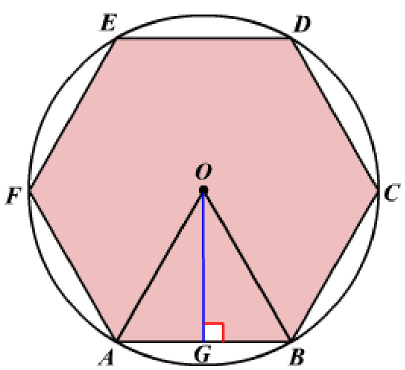
Eudoxus and the "method of exhaustion"

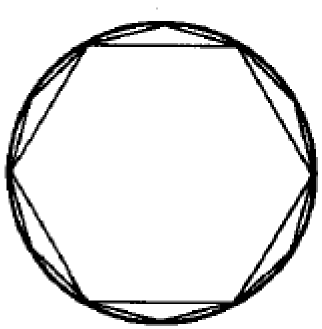
- Eudoxus seems to have been the first person to define our modern notion of a "limit" which he referred to as "the method of exhaustion."
- In modern notation, $\lim_{n\to\infty} M(1-r)^n = 0$, for $0<r<1$.

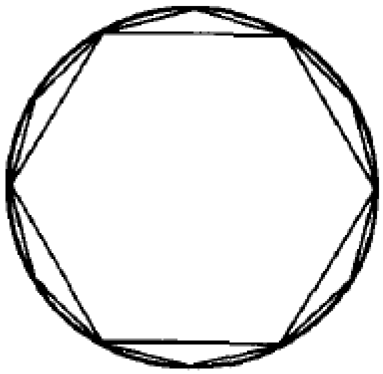
If from any magnitude there be subtracted a part not less than its half, and if from the remainder one again subtracts not less than its half, and if this process of subtraction is continued, ultimately there will remain a magnitude less than any preassigned magnitude of the same kind.[8]

Ratios of areas of circles

- The argument of Eudoxus involved finer and finer inscribed polygons.
- The ratio of the areas of regular polygons is proportional to the squares of the "radius" of the polygons. Hence the theorem.

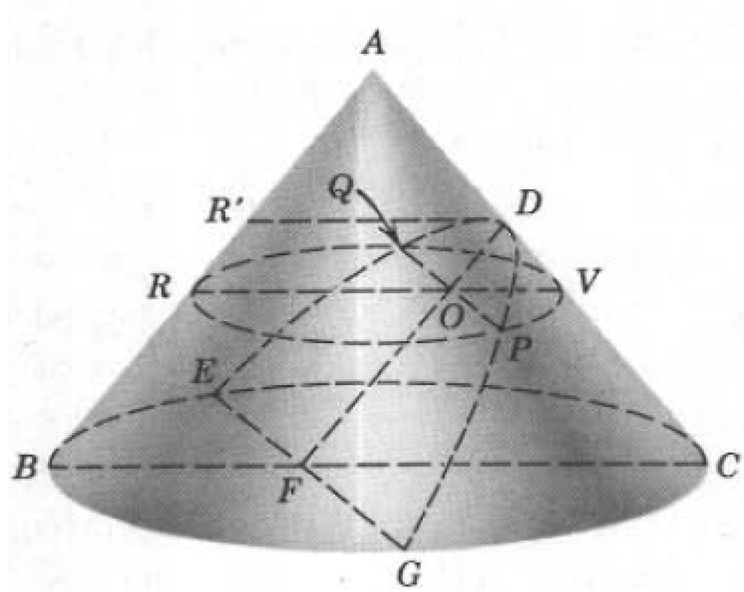


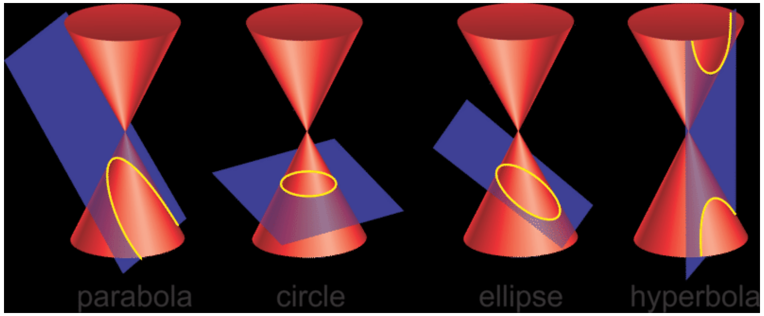




The conic sections

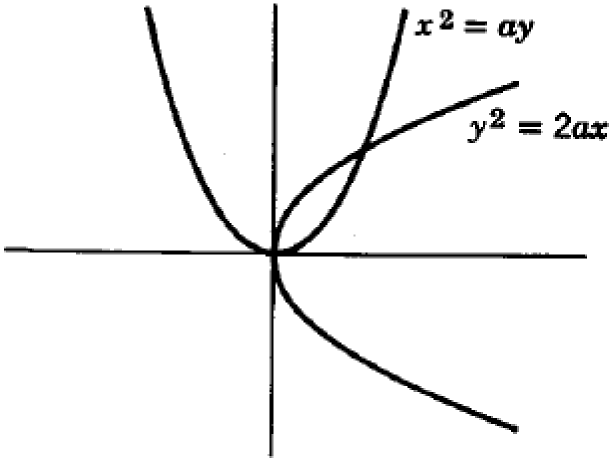
- The brothers Menaechmus and Dinostratus were taught by Eudoxus.
- Menaechmus is credited with the discovery of the conic sections: the parabola, the ellipse, and the hyperbola.





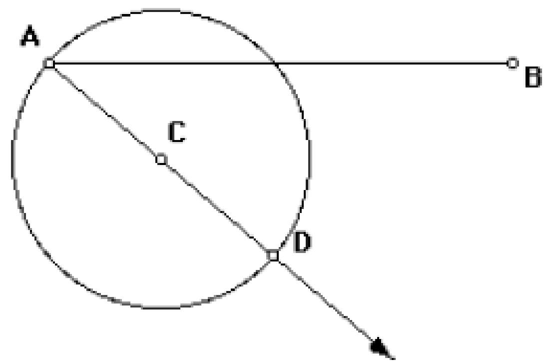
The duplication of the cube

- Menaechmus knew how to solve the problem of the duplication of the cube if one is permitted to use parabolas.
- The intersection of the two parabolas allows the construction of the cube root of 2.

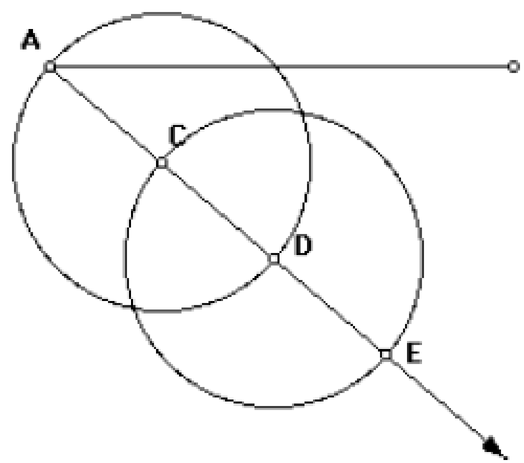


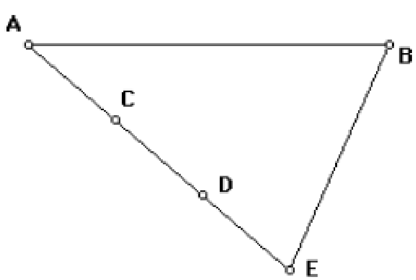
Trisecting a line segment

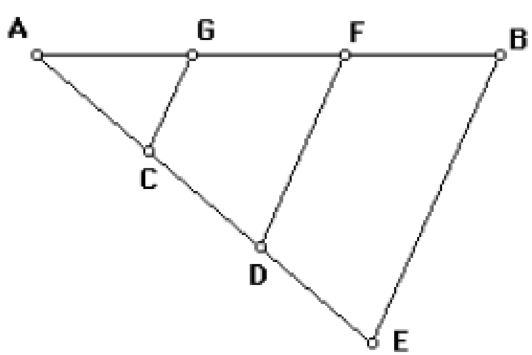
- 1. Draw any ray AC.
- 2. Draw circle of radius AC centered at C.



- 3. Draw another circle of radius AC centered at D.

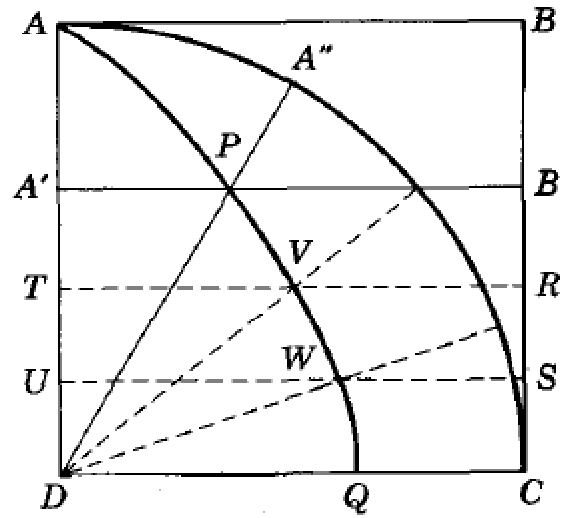






Trisecting angles

- Dinostratus studied the squaring of the circle using what has been called the "trisectrix" of Hippias.
- This is the locus intersection of side DA rotating clockwise and side AB dropping to coincide with DC.
- To trisect PDC, trisect the line segments B'C and A'D at R,S,T,U.
- Join T to R and U to S to trisect the given angle.



Squaring the circle using the trisectrix

- The equation of the trisectrix is $\pi r \sin\theta = 2a\theta$.
- As $\theta \to 0$, r tends to $2a/\pi$.
- Thus DQ is $2a/\pi$.
- From this, the squaring of the circle is easily determined.