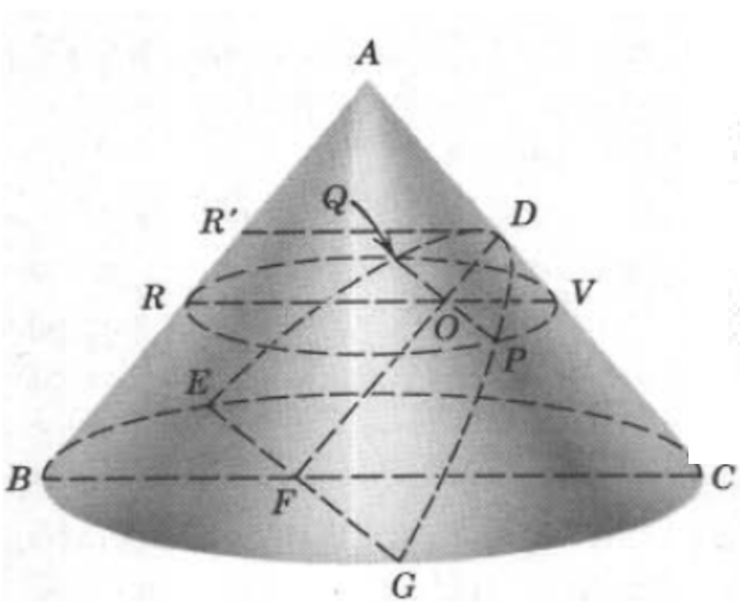
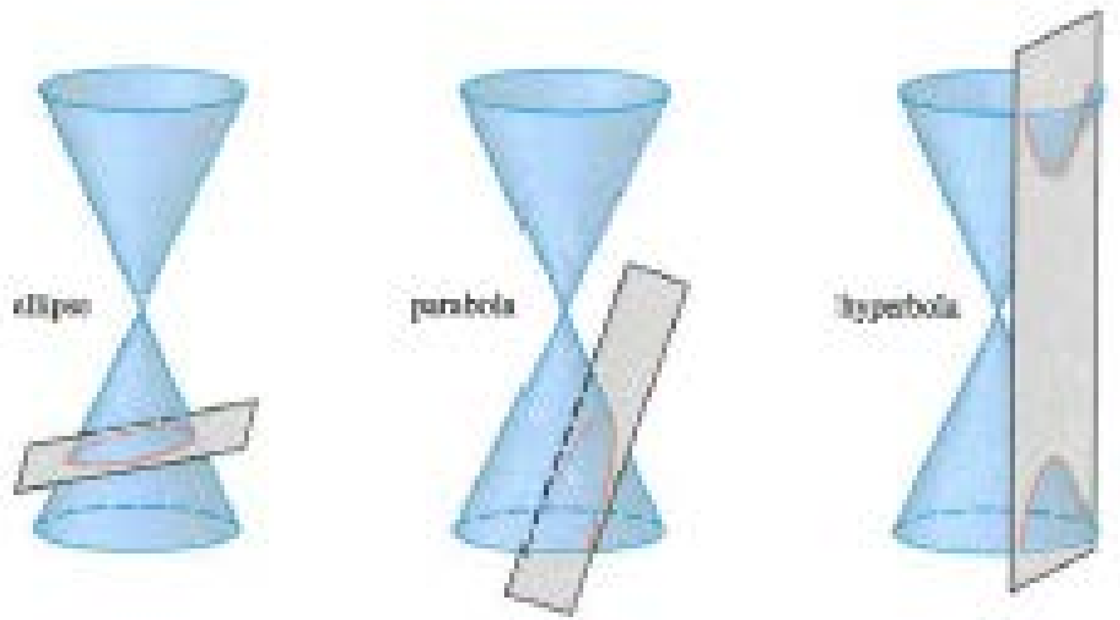
Appolonius and conic sections



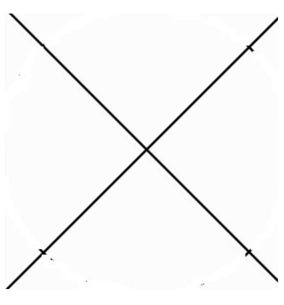


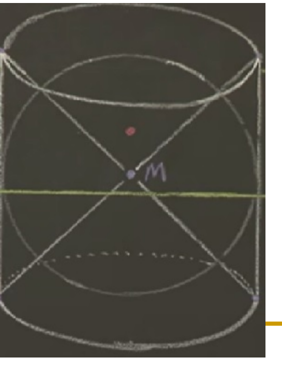
Appolonius of Perga

- Appolonius of Perga should be distinguished from Appolonius of Tyana who was a later philosopher.
- Appolonius seems to have lived between 262 BCE and 190 BCE and his famous for his work "The Conics".
- The work survives because the Arabic mathematician Thabit ibn Qurra had preserved it and translated it into Arabic.
- In 1710, Edmund Halley translated this into Latin.

The cone and its sections

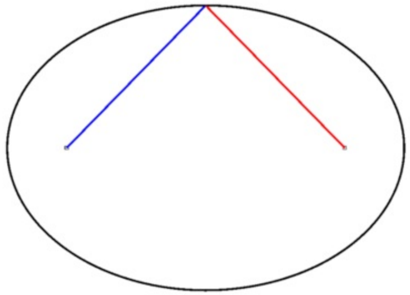
- Just as Archimedes was preoccupied with sections of the sphere, Appolonius noticed that the parabola, the ellipse and hyperbola can all be seen as planar sections of the cone.
- The cone was seen as the revolution of the triangle determined by the diagonals of a rectangle.

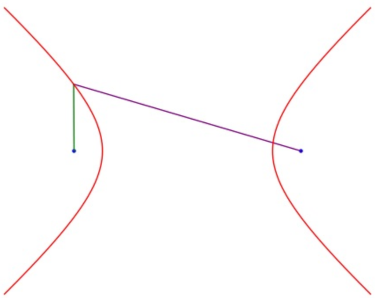


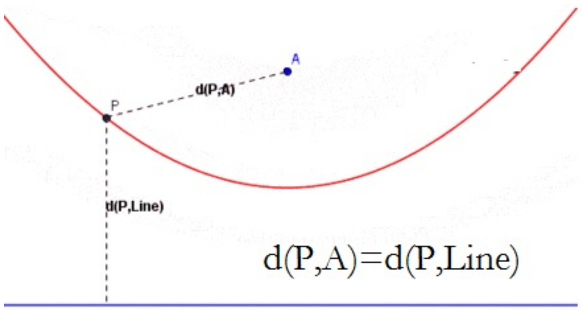


Foci of conics

- Today, it is recognized that the focal points of conic sections play an important role in describing the conic.
- Yet, Appolonius had no specific name for them though he referred to them indirectly.

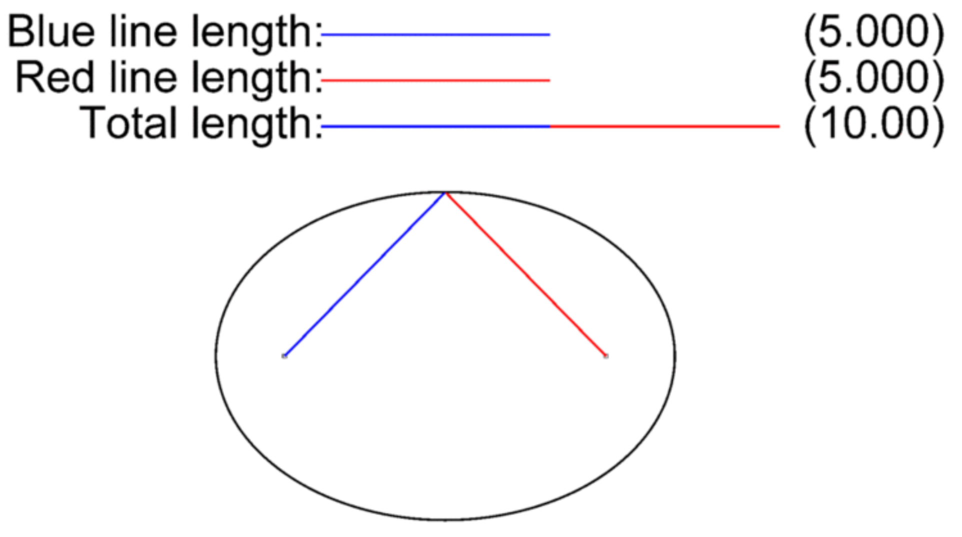






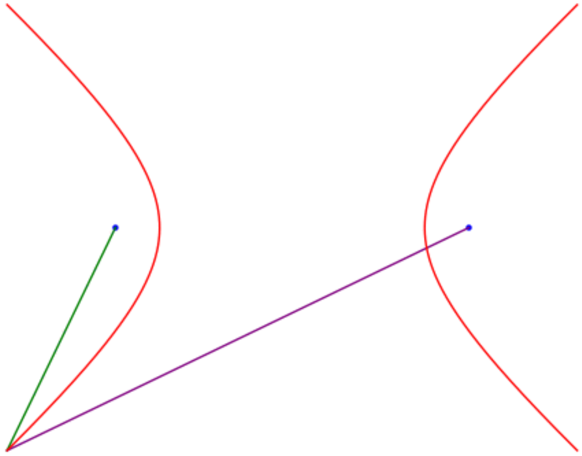
The ellipse

- The ellipse is the locus of points with sum of distance to the two foci is fixed.



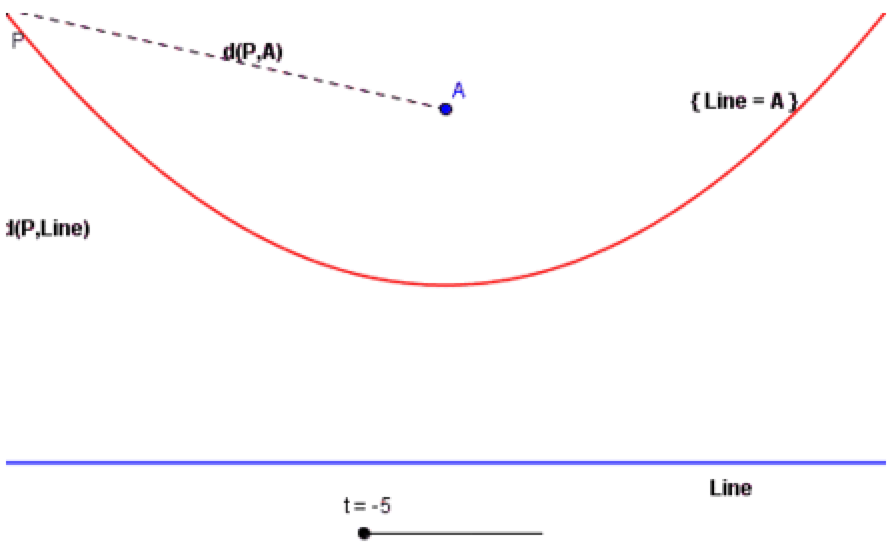
The hyperbola

- The hyperbola is the locus of points whose difference of the distances to the two foci is constant.



The parabola

- The parabola is the locus of points equidistant to a focal point and a given line.



Equations for the conic sections

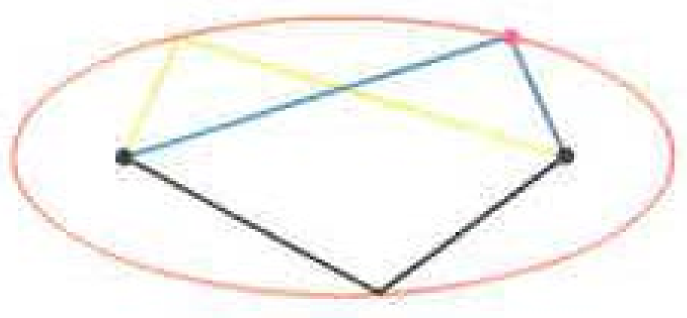
- Appolonius did not write down the formulas for the conic sections as we describe them today.
- This was done later after Descartes introduced what we now refer to as the x-y axis, or Cartesian co-ordinates.
- In the 17th century, Kepler and Galileo both realized the importance of the study of conic sections to describe planetary motion.

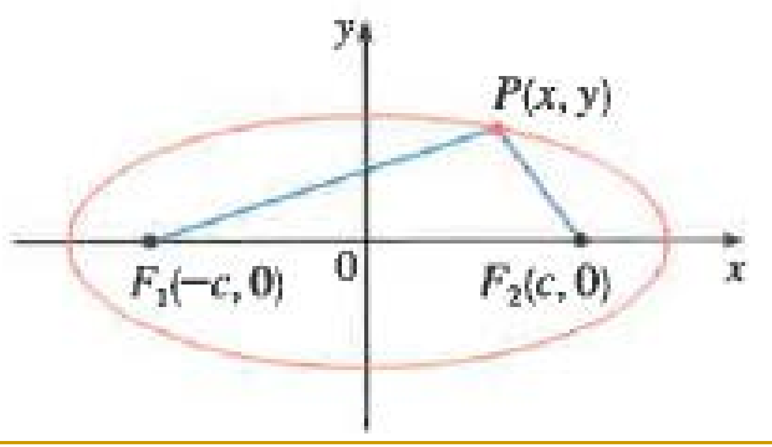
Kepler trying to explain his discovery

- In the 17th century, Kepler discovered that planets in our solar system follow elliptical orbits with the sun as one of the focal points.

Equation for an ellipse

- An ellipse is determined by two fixed points called foci and a distance d.
- It is the locus of points P=(x,y) in the plane such that the sum of the distances PF_1 and PF_2 is constant and equal to 2a.

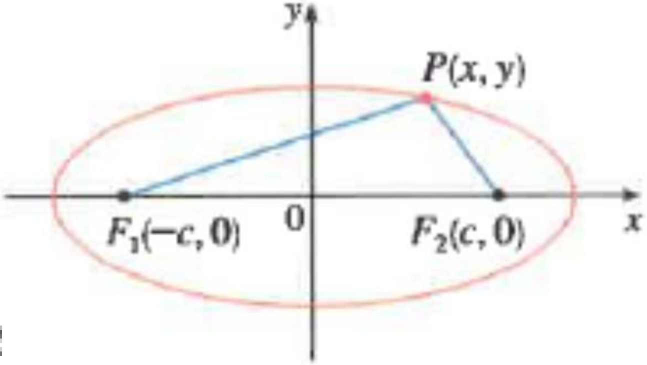




The derivation in Cartesian co-ordinates

- We choose a convenient co-ordinate system:

$$|PF_1| + |PF_2| = 2a$$



$$\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a$$

$$\sqrt{(x-c)^2 + y^2} = 2a - \sqrt{(x+c)^2 + y^2}$$

.

Squaring both sides, we have

$$x^2 - 2cx + c^2 + y^2 = 4a^2 - 4a\sqrt{(x+c)^2 + y^2} + x^2 + 2cx + c^2 + y^2$$

The final result

- We get:

Squaring both sides, we have

$$x^2 - 2cx + c^2 + y^2 = 4a^2 - 4a\sqrt{(x+c)^2 + y^2} + x^2 + 2cx + c^2 + y^2$$

which simplifies to $$a\sqrt{(x+c)^2 + y^2} = a^2 + cx$$

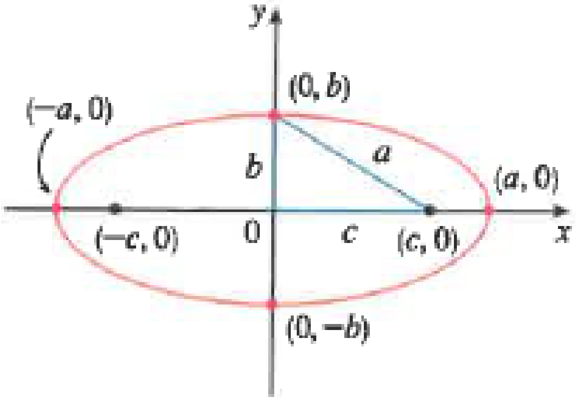
We square again:

$$a^2(x^2 + 2cx + c^2 + y^2) = a^4 + 2a^2cx + c^2x^2$$

which becomes $$(a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2)$$

$$b^2 = a^2 - c^2.$$

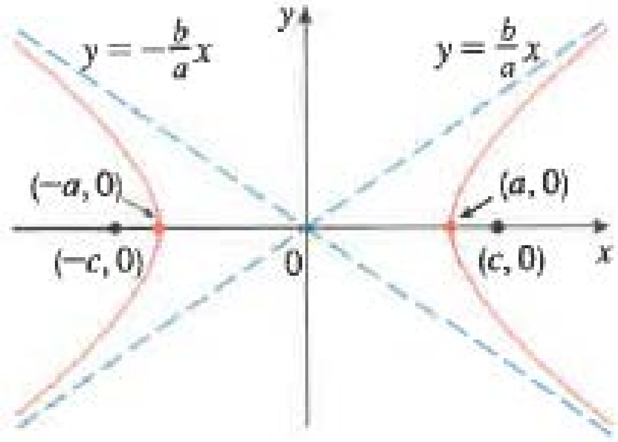
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



The equation for the hyperbola

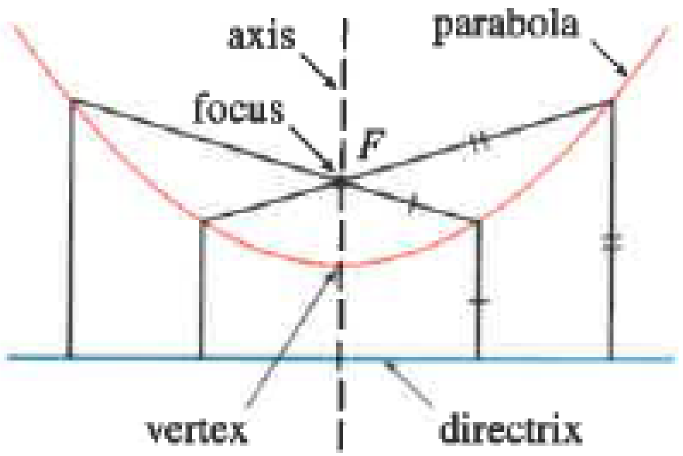
- The derivation of the formula for the hyperbola is similar and is left as an exercise.
- The final answer is:

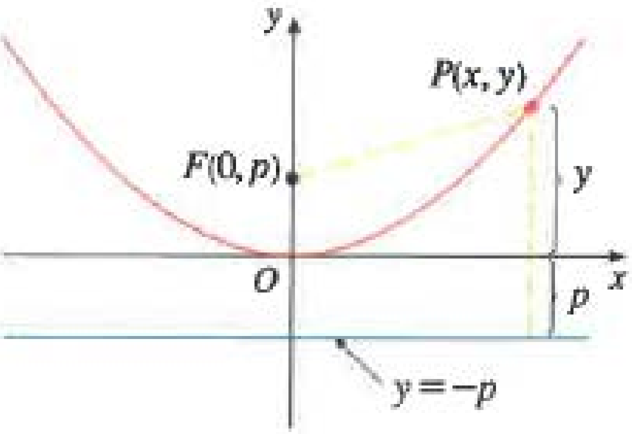
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



The equation for the parabola

- We choose our line and axis of symmetry conveniently.





$$|PF| = \sqrt{x^2 + (y-p)^2}$$

$$\sqrt{x^2 + (y-p)^2} = |y+p|$$

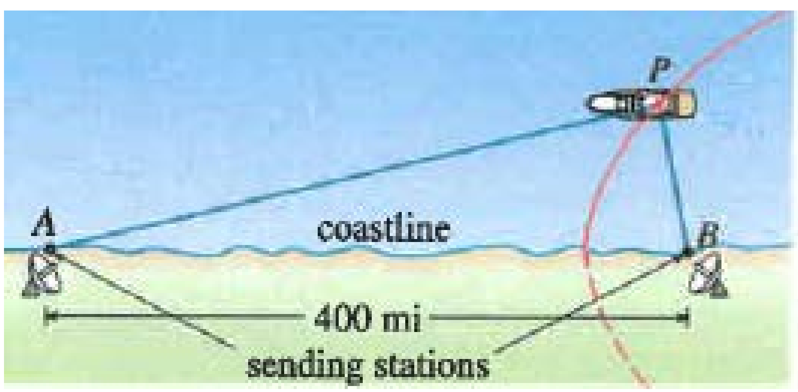
$$x^2 + (y-p)^2 = |y+p|^2 = (y+p)^2$$

$$x^2 + y^2 - 2py + p^2 = y^2 + 2py + p^2$$

$$x^2 = 4py$$

Pure versus applied mathematics

- Much of the work of Appolonius was motivated by an aesthetic desire to understand symmetry and beauty of the conic sections.
- It was only after about 1800 years that his work occupies a central place in physics, and especially astronomy, in describing the motion of the planets.
- Galileo discovered that the path of a projectile follows the path of a parabola.
- Today, we use hyperbolic curves to determine positions of aircraft and ships using radar.



The time difference in receiving the radio signals tells us the ship location is on a particular hyperbola. We use one more radio signal to determine another hyperbola and the intersection gives the position. GPS works on a similar principle.

The music of the spheres