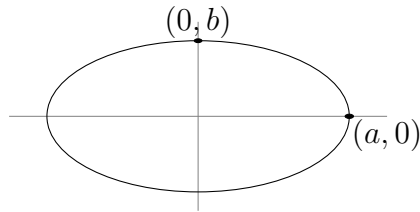# MATH 381: Assignment 2 (due: February 7, 2020)

1. Show that if d divides n and a is a natural number greater than 1, then $a^d - 1$ divides $a^n - 1$. Deduce that if $2^n - 1$ is prime, then n is prime. [Hint: write $n = de$ and consider $1 + a^e + a^{2e} + \cdots + a^{(d-1)e}$.]

2. Using calculus, show that the area of the ellipse with semi-major axis a and semi-minor axis b is πab:

3. Using Cartesian co-ordinates, derive the equation of the hyperbola with foci at $(-c, 0)$ and $(c, 0)$ such that $|PF_1| - |PF_2| = \pm 2a$:
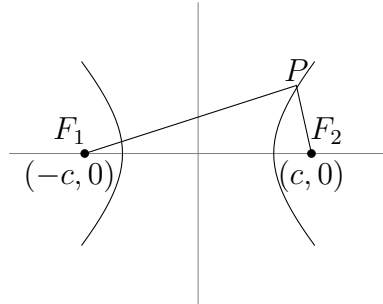
4. Define the Möbius function μ as follows: $\mu(1) = 1$, $\mu(n) = 0$ if n is divisible by the square of a prime and $(-1)^r$ if n is a product of r distinct primes. Show that if $n > 1$, then

$$\sum_{d|n} \mu(d) = 0.$$

5. Write a short essay (minimum 1 page; maximum 2 pages, typed in 12 point font, double spaced) discussing aspects of aesthetics in the works of Euclid, Archimedes and Appolonius.