MATH 381: Assignment 3 (due: February 28, 2020)

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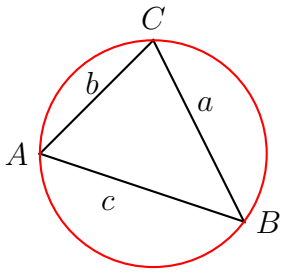
1. Show that a triangle with side lengths 3, 4 and 5 units is necessarily a right angled triangle.

2. Two numbers are called *amicable* if each is equal to the sum of the **proper** divisors of the other. If p, q, r are distinct primes of the form $3 \cdot 2^{n-1} - 1, 3 \cdot 2^n - 1$ and $9 \cdot 2^{2n-1} - 1$ respectively, show that
$$M = 2^n pq, \quad N = 2^n r,$$
are amicable numbers. (This is a result of Thabit Ibn-Qurra proved around 900 CE.)

3. Show that the diameter D of the circumscribed circle of a triangle ABC with angles A, B, C and opposite sides a, b, c (see diagram) satisfies $D = \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.



4. Suppose we have a sequence $a_0, a_1, \ldots$ that satisfies a recurrence relation
$$a_n = Aa_{n-1} + Ba_{n-2}, \qquad n \geq 2,$$
where A, B are real numbers and $B \neq 0$. Suppose that $A^2 + 4B \neq 0$. Let α and β be roots of the quadratic equation
$$x^2 - Ax - B = 0.$$
Show that
$$a_n = C\alpha^n + D\beta^n,$$
for certain real numbers C and D. What happens if $A^2 + 4B = 0$? Derive a formula for a_n in this case also.

5. Write a short essay (minimum 1 page; maximum 2 pages, typed in 12 point font, double spaced) discussing how the decimal system influenced the growth of mathematics.