MATH 401/801: Assignment 3

Math 401: Do any eight; Math 801: Do all ten.
Due: November 21, 2018

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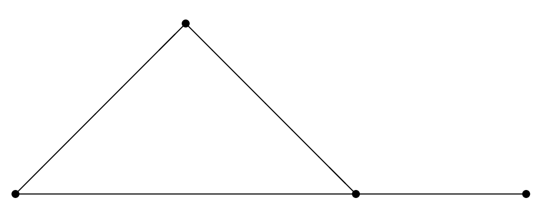
1. Let X be a graph with chromatic number 2. Show that it is bipartite.

2. Six different television stations are applying for channel frequencies and no two stations can use the same frequency if they are within 150 miles of each other. If the distances between the stations A, B, C, D, E and F are given by the matrix below, find the minimal number of frequencies needed.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{pmatrix} - & 85 & 175 & 200 & 50 & 100 \\ 85 & - & 125 & 175 & 100 & 160 \\ 175 & 125 & - & 100 & 200 & 250 \\ 200 & 175 & 100 & - & 210 & 220 \\ 50 & 100 & 200 & 210 & - & 100 \\ 100 & 160 & 250 & 220 & 100 & - \end{pmatrix} \end{array}.$$

3. Prove that the sum of the coefficients of the chromatic polynomial of a graph X is zero unless X has no edges.

4. Compute the chromatic polynomial of the graph



5. The **join** of two graphs X and Y is defined as the graph obtained by joining every vertex of X to every vertex of Y. We denote this graph by $X \vee Y$. Show that $\chi(X \vee Y) = \chi(X) + \chi(Y)$.

6. The wheel graph is $K_1 \vee C_n$. That is, the wheel graph is the cycle graph together with a vertex at the 'center' which is connected to all the vertices of C_n. Determine the chromatic polynomial of the wheel graph.

7. Let $p_X(\lambda)$ be the chromatic polynomial of a connected graph X with n vertices. Show that

$$|p_X(\lambda)| \leq \lambda(\lambda - 1)^{n-1}$$

if $n \geq 3$.

8. If $p_X(\lambda)$ is the chromatic polynomial of a graph X, show that we can write it as $\lambda^c f(\lambda)$ where $f(0) \neq 0$ and c is the number of connected components of X.

9. (a) If X is a simple graph, show that its chromatic number satisfies

$$\chi(X) \leq 1 + \sqrt{2e(n-1)/n}$$

where e is the number of edges and n is the number of vertices of X.

(b) Show that the chromatic number of a graph X is

$$\leq \frac{1 + \sqrt{8e+1}}{2}.$$

This result is slightly sharper than (a).

10. Let X_n be the graph with vertex set $\{1, 2, ..., 2n\}$ with the adjacency relation given by (i, j) is an edge if and only if i and j have a common prime divisor. Show that the chromatic number of X_n is at least n.