MATH 382: Assignment 1 (due: September 26, 2019)

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1. Use the Euclidean algorithm to find all integers x and y such that
$$42823x + 6409y = 17.$$
2. If $m = p_1^{a_1} \cdots p_k^{a_k}$ and $n = p_1^{b_1} \cdots p_k^{b_k}$ are the respective unique factorizations, show that
$$\text{gcd}(m, n) = p_1^{\min\{a_1,b_1\}} \cdots p_k^{\min\{a_k,b_k\}},$$
and
$$\text{lcm}(m, n) = p_1^{\max\{a_1,b_1\}} \cdots p_k^{\max\{a_k,b_k\}}.$$
3. Show that for all natural numbers $n \geq 1$,
$$1 \cdot 1! + 2 \cdot 2! + \cdots n \cdot n! = (n + 1)! - 1.$$
4. Find all integers x such that
$$11111x \equiv 10 \pmod{78787}.$$
5. Show that for all natural numbers $N \geq 1$,
$$\sum_{n=1}^{N} \frac{1}{n(n+1)} = 1 - \frac{1}{N+1}.$$
6. Let p_n denote the n-th prime. Show that for $n \geq 1$,
$$p_n < 2^{2^n}.$$
7. Show that if $n|2^n - 1$, then $n = 1$.
8. Show that for any natural number $n > 1$, the number
$$S := 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$
is not a natural number. [Hint: consider k such that $n/2 < 2^k \leq n$ and let d be the lcm of all the numbers 1, 2, ..., n except for 2^k and analyze dS.]
9. Let $d = \text{gcd}(m, n)$. Show that
$$\text{gcd}(a^m - 1, a^n - 1) = a^d - 1,$$
for any natural number $a > 1$.
10. The Fibonacci sequence is defined as follows. $F_1 = 1$, $F_2 = 1$ and for $n \geq 3$, $F_n = F_{n-1} + F_{n-2}$. Show that
    (a) the gcd of F_n and F_{n-1} is 1 for $n \geq 2$;
    (b) $F_{n+m} = F_{n-1}F_m + F_nF_{m+1}$ for $n \geq 2$ and $m \geq 1$; [Hint: induct on m.]
    (c) $\text{gcd}(F_n, F_m) = F_{\text{gcd}(n,m)}$.