MATH 382: Assignment 3 (due: November 22, 2019)

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1. Show that every prime number p can be written as $p = x^2 + y^2 + z^2 + 4w^2$ in integers.

2. Find the simple continued fraction expansions of $17/3$ and $3/17$.

3. Show that
$$[1, 1, 1, ...] = \frac{1+\sqrt{5}}{2}.$$

4. Evaluate
$$[1, 3, \overline{1, 2}].$$

5. Find the continued fraction of $\sqrt{5}$.

6. Show that $[d, \overline{2d}]$ is the continued fraction expansion of $\sqrt{d^2+1}$, for any natural number d.

7. Given natural numbers b, c, d with $c > d$, show that
$$[a, c] < [a, d]$$
but
$$[a, b, c] > [a, b, d].$$

8. Let $a_0, a_1, ..., a_n$ and c be positive real numbers. Prove that
$$[a_0, a_1, ..., a_{n-1}, a_n] > [a_0, a_1, ..., a_{n-1}, a_n + c]$$
if n is odd. [Hint: induct.]

9. Find the fundamental solution of
$$x^2 - 5y^2 = 1.$$
More generally, find the fundamental solution of
$$x^2 - (d^2+1)Y^2 = 1.$$

10. Let $\alpha \notin \mathbb{Q}$. Show that there are infinitely many natural numbers Q_i tending to infinity and rational numbers p_i/q_i such that
$$|q_i\alpha - p_i| \leq \frac{1}{Q_i}.$$
[Hint: apply successively Dirichlet approximation theorem.]