Math 892: Assignment 1 (due: February 1, 2019)

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1. Compute the Fourier transform of the function $f \in L^1(\mathbb{R})$ given by $f(x) = 1$ if $x \in [-1, 1]$ and zero otherwise. Show that $\widehat{f} \in L^2(\mathbb{R})$ but not in $L^1(\mathbb{R})$.

2. Prove that
$$\int_{-\infty}^{\infty} \frac{e^{itx} dt}{1+t^2} = \pi e^{-|x|}.$$

3. Find the Fourier transform of
$$f(x) = \frac{x}{(1+x^2)^2}.$$

4. Solve for f:
$$\int_{-\infty}^{\infty} f(x-t) e^{-|t|} dt = \frac{4}{3} e^{-|x|} - \frac{2}{3} e^{-2|x|}.$$

5. In a Hilbert space, show that the maps $x \mapsto (x, y)$ for fixed y and the map $x \mapsto ||x||$ are continuous maps.

6. For functions $f \in L^1(\mathbb{R}^n)$, we define the Fourier transform
$$\hat{f}(t) = (2\pi)^{-n/2} \int_{\mathbb{R}^n} f(x) e^{-i(x,t)} dx,$$
where (x, t) denotes the usual inner product and dx is the Lebesgue measure on $\mathbb{R}^n$. If $f(x) = e^{-|x|^2/2}$, show that $\hat{f} = f$. (Here $x = (x_1, ..., x_n)$ and $|x|^2 = x_1^2 + \cdots + x_n^2$.)

7. Let
$$F_N(x) = \sum_{|n| \le N} \left(1 - \frac{|n|}{N}\right) e^{2\pi i n x}.$$
Show that
$$F_N(x) = \frac{1}{N} \left| \sum_{n=0}^{N-1} e^{2\pi i n x} \right|^2 = \frac{1}{N} \left(\frac{\sin \pi N x}{\sin \pi x} \right)^2.$$