1. Let \( w \in \mathbb{C} \) be a fixed complex number with \(|w| < 1\). Let

\[
f(z) = \frac{z - w}{1 - wz}.
\]

Show that \( f \) is regular in \(|z| \leq 1\) and calculate \( f(w) \) and \( f'(w) \).

2. Find the radius of convergence of

\[
\sum_{n=1}^{\infty} n! \frac{z^n}{n^n}.
\]

3. Prove the following discrete version of integration by parts: For any two sequences of numbers \( a_n, b_n \),

\[
S_N := \sum_{n=1}^{N} a_n b_n = a_N B_N - \sum_{n=1}^{N-1} B_n (a_{n+1} - a_n),
\]

where

\[
B_n = \sum_{j=1}^{n} b_j.
\]

Using this result, show that the power series

\[
\sum_{n=1}^{\infty} \frac{z^n}{n}
\]

converges for every complex number \( z \) with \(|z| = 1\), and \( z \neq 1 \).

4. Show that the series

\[
\sum_{n=1}^{\infty} \frac{z^n}{n(n+1)}
\]

converges for all \(|z| \leq 1\). What is its radius of convergence? Does the series

\[
\sum_{n=1}^{\infty} z^n
\]

converge for any \( z \) with \(|z| = 1\)?

5. Let \( n \) be a non-negative integer. Define for \( \alpha \in \mathbb{C} \), the binomial coefficient

\[
\binom{\alpha}{n} = \frac{\alpha(\alpha - 1) \cdots (\alpha - n + 1)}{n!}.
\]

Show that the series

\[
\sum_{n=0}^{\infty} \binom{\alpha}{n} z^n
\]

converges for \(|z| < 1\).