Math 892: Assignment 3 (due: March 29, 2019)

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1. Determine the number of zeros of the polynomial

$$2z^5 - 6z^2 + z + 1 = 0$$

in the annulus $1 \leq |z| \leq 2$.

2. Let f be a meromorphic function and A its set of poles. Suppose it has a finite number of poles all of which lie in the upper half-plane (that is, have positive imaginary part). Suppose there is a constant $K > 0$ such that for some fixed $\delta > 1$, we have

$$|f(z)| \leq K/|z|^\delta,$$

for $|z|$ sufficiently large. Show that

$$\int_{-\infty}^{\infty} f(x)dx = 2\pi i \sum_{a \in A} \operatorname{Res}(f; a).$$

3. Show that

$$\int_{-\infty}^{\infty} \frac{\cos x}{x^2+1} dx = \frac{\pi}{e}.$$

4. Let $f(z)$ have a simple pole at $z = 0$. Let $C(\epsilon)$ be the semicircular arc from $-\epsilon$ to ϵ of radius $\epsilon > 0$. Show that

$$\lim_{\epsilon \to 0} \int_{C(\epsilon)} f(z)dz = -\pi i \operatorname{Res}(f; 0).$$

[Note: $C(\epsilon)$ is not a closed path.]

5. Suppose α is a complex number, $|\alpha| \neq 1$. Compute

$$\int_0^{2\pi} \frac{d\theta}{1 - 2\alpha\cos\theta + \alpha^2}.$$