1. Calculate the eigenvalues of the adjacency matrix of the graph on four vertices
\( \circ \rightarrow \circ \circ \rightarrow \circ \). Is this a bipartite graph? Justify your answer.

2. What is a tree? Show that a tree on \( n \) vertices has \( n - 1 \) edges.

3. If \( T \) is a tree on \( n \) vertices with a vertex of degree \( k \), show that \( T \) has at least \( k \) leaves (that is, vertices of degree one).

4. Let \( X \) be a simple graph with \( n \) vertices \( v_1, v_2, ..., v_n \) where \( n \geq 2 \). Let \( A \) be its adjacency matrix with eigenvalues \( \lambda_1, \lambda_2, ..., \lambda_n \). Show that there are real numbers \( c_1, c_2, ..., c_n \) such that the number of paths of length \( r \) from \( v_1 \) to \( v_2 \) is given by

\[
c_1 \lambda_1^r + c_2 \lambda_2^r + \cdots + c_n \lambda_n^r,
\]

for any \( r \geq 1 \).