Math 892: Midterm test - February 15, 2019

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1. Define the Fourier transform of a function $f \in L^1(\mathbb{R})$. Calculate the Fourier transform of $f(x) = e^{-x^2-2x}$. [Hint: $(x+1)^2 = x^2 + 2x + 1$.]

2. Let $f(z) = u(x, y) + iv(x, y)$ be a complex valued function with $u(x, y), v(x, y)$ real-valued and $i = \sqrt{-1}$. Define $\frac{\partial f}{\partial z}$ and $\frac{\partial f}{\partial \bar{z}}$ by setting

$$\frac{\partial f}{\partial z} := \frac{1}{2}\left(\frac{\partial f}{\partial x} - i\frac{\partial f}{\partial y}\right),$$

$$\frac{\partial f}{\partial \bar{z}} := \frac{1}{2}\left(\frac{\partial f}{\partial x} + i\frac{\partial f}{\partial y}\right).$$

Show that the Cauchy-Riemann equations are equivalent to

$$\frac{\partial f}{\partial \bar{z}} = 0.$$

Moreover, show that if $f(z)$ is holomorphic, then

$$f'(z) = \frac{\partial f}{\partial z}.$$