SIGN CHANGES OF FOURIER COEFFICIENTS
OF HALF-INTEGRAL WEIGHT CUSP FORMS

JABAN MEHER∗ and M. RAM MURTY†

Department of Mathematics
Queen’s University, Kingston
ON, Canada, K7L 3N6
∗jaban@mast.queensu.ca
†murty@mast.queensu.ca

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We prove a quantitative result for the number of sign changes of the Fourier coefficients of
half-integral weight cusp forms in the Kohnen plus space, provided the Fourier coefficients
are real numbers.

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1. Introduction

Let $k$ be a positive integer. Suppose that $f$ is a modular form of weight $k + 1/2$
on $\Gamma_0(4)$. The Shimura correspondence defined in [12] maps $f$ to a modular form
$F$ of integral weight $2k$. In addition, if $f$ is an eigenform of the Hecke operator
$T_{k+1/2}(p^2)$, then $F$ is also an eigenform of the Hecke operators $T_{2k}(p)$ with the
same eigenvalue. For more on half-integral weight modular forms, see [12]. Let the
Fourier expansion of $f$ be given by

$$f(z) = \sum_{n=0}^{\infty} a(n)n^{k-\frac{1}{2} - \frac{1}{4}}e^{2\pi intz}. \quad (1)$$

Also, assume that the Fourier coefficients $a(n)$ are real numbers. Kohnen [6] proved
that for any half-integral weight cusp form, the sequence $a(tn^2)$ for a fixed $t$ square-
free has infinitely many sign changes. However, Kohnen does not prove any quanti-
tative result on the number of sign changes in that paper. Recently, Kohnen, Lau
and Wu [7] have proved some quantitative results on the number of sign changes in
the sequence $a(tn^2)$, where $t$ is a fixed square-free positive integer. If $f$ is an eigen-
form, then recently, Hulse, Kiral, Kuan and Lim [3] have proved that the sequence
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\(a(t)\), where \(t\) is running over all square-free integers, changes sign infinitely often. If \(f\) is in the Kohnen plus space, that is, \(a(n) = 0\) when \((-1)^k n \equiv 2, 3 \pmod{4}\), Bruinier and Kohnen [1] make the following conjectures:

\[
\lim_{x \to \infty} \frac{|\{n \leq x : a(n) < 0\}|}{|\{n \leq x : a(n) \neq 0\}|} = \frac{1}{2},
\]

and

\[
\lim_{x \to \infty} \frac{|\{|d| \leq x : |d| \text{ fundamental discriminant}, a(|d|) < 0\}|}{|\{|d| \leq x : |d| \text{ fundamental discriminant}, a(|d|) \neq 0\}|} = \frac{1}{2}.
\]

The proofs of the above conjectures seem to be out of reach as of now. However, in this article we prove a quantitative result on the number of sign changes in the sequence \(a(n)(n \geq 1)\). In general, one may ask about a quantitative result on the number of sign changes for any real sequence. In fact, under certain conditions, we are able to prove a quantitative result on number of sign changes for any real sequence.

More precisely, we prove the following result.

**Theorem 1.1.** Let \(a(n)(n \geq 1)\) be a sequence of real numbers satisfying \(a(n) = O(n^\alpha)\) such that

\[
\sum_{n \leq x} a(n) \ll x^\beta,
\]

and

\[
\sum_{n \leq x} a(n)^2 = cx + O(x^\gamma),
\]

where \(\alpha, \beta, \gamma\) and \(c\) are non-negative constants. If \(\alpha + \beta < 1\), then for any \(r\) satisfying \(\max\{\alpha + \beta, \gamma\} < r < 1\), the sequence \(a(n)(n \geq 1)\) has at least one sign change for \(n \in (x, x + x^r]\). In particular, the sequence \(a(n)(n \geq 1)\) has infinitely many sign changes and the number of sign changes for \(n \leq x\) is \(\gg x^{1-r}\) for sufficiently large \(x\).

The above theorem can be applied in a very general context. Here, we focus our attention on the sequence of Fourier coefficients of cusp forms of half-integral weight on \(\Gamma_0(4)\). In a later paper [9], we investigate several applications to the theory of \(L\)-functions attached to automorphic representations.

**Theorem 1.2.** Let \(k \geq 2\) be an integer. Assume that

\[
f(z) = \sum_{(-1)^n \equiv 0, 1 \pmod{4}} a(n) n^{k-1/2} e^{2\pi i nz}
\]

is an eigenform of weight \(k + 1/2\) on \(\Gamma_0(4)\) in the Kohnen’s plus space \(S_{k+1/2}^+ (\Gamma_0(4))\) such that the cusp form \(F\) associated to \(f\) under the Shimura correspondence is an eigenform. If the coefficients of \(f\) are real numbers, then there exists \(0 < \delta < 1\) such that the sequence \(a(n) (n \geq 1)\) has at least one sign change for \(n \in (x, x + x^\delta]\).
for sufficiently large $x$. In particular, the number of sign changes for $n \leq x$ is $\gg x^{1-\delta}$. We prove that one may choose $\delta$ as any number greater than $\frac{1}{3}$. If we assume the Ramanujan conjecture, then one may choose $\delta$ as any number greater than $\frac{4}{3}$.

2. Preparatory Results

One of key ingredients in the proof is Waldspurger’s result [13]. For an eigenform $f \in S_{k+1/2}^+(\Gamma_0(4))$, the Waldspurger formula is more explicit as proved by Kohnen and Zagier [8]. For a fundamental discriminant $D$, the $L$-series $L_D(s)$ is defined as

$$L_D(s) = \sum_{n \geq 1} \left( \frac{D}{n} \right) n^{-s}.$$  

If $D = l^2D_0$, where $D_0$ is a fundamental discriminant, then $L_D(s)$ is defined as $L_{D_0}(s)$ times a finite Euler product over the prime divisors of $l$. If $D \not\equiv 0, 1 \pmod{4}$, then $L_D(s)$ is defined to be identically $0$. Thus, we have defined $L_D(s)$ for all $D \neq 0$.

For an arbitrary integer $D \neq 0$, $L(f, D, s)$ is defined as the convolution of $L(f, s)$ and $L_D(s)$, where

$$L(f, s) = \sum_{n \geq 1} \frac{a(n)}{n^s}.$$  

More precisely,

$$L(f, D, s) = \sum_{n \geq 1} \left( \frac{D}{n} \right) \frac{a(n)}{n^s}.$$  

We state the following result of Kohnen and Zagier [8, Corollary 4].

**Lemma 2.1.** Let

$$f(z) = \sum_{(-1)^{1/2}n=1,\pmod{4}} a(n) n^{k/2} \frac{z^k}{k!} e^{2\pi inz}$$

be an eigenform of weight $k+1/2$ on $\Gamma_0(4)$ in the Kohnen’s plus space $S_{k+1/2}^+(\Gamma_0(4))$ such that the cusp form $F$ associated to $f$ under the Shimura correspondence is an eigenform. Then one has

$$\frac{|a(n)|^2}{\langle f, f \rangle} = \frac{(k-1)!}{\pi^{k/2} \langle F, F \rangle} L(F, (-1)^k n, k),$$

where $\langle f, f \rangle$ and $\langle F, F \rangle$ are the Petersson inner products in the spaces of half-integral weight and integral weight cusp forms, respectively.

We review some more facts about half-integral weight modular forms. Assume that $h(z) = \sum_{n \geq 1} c(n) e^{2\pi inz}$ is a cusp form of weight $k + 1/2$ on $\Gamma_0(4)$. Let

$$D_h^*(s) = \pi^{-s} \Gamma(s) \sum_{n \geq 1} c(n) n^{-s}.$$
Then $D_h^\ast(s)$ has an analytic continuation to the whole complex plane $\mathbb{C}$ and it satisfies the functional equation

$$D_h^\ast(k + 1/2 - s) = D_h^\ast(W_4(s),$$

where

$$h \mid W_4 = (-2iz)^{-k-1/2}h(-1/4z) = \sum_{n \geq 1} c^\ast(n)e^{2\pi inz}$$

is a cusp form of weight $k + 1/2$ on $\Gamma_0(4)$ [5, p. 430]. If $c(n) = a(n)n^{k/2-1/4}$ and $c^\ast(n) = a^\ast(n)n^{k/2-1/4}$, let $M_1(s) = \sum_{n \geq 1} a(n)n^{-s}$ and $M_2(s) = \sum_{n \geq 1} a^\ast(n)n^{-s}$. Then the above functional equation becomes

$$\pi^{-(s+\frac{k}{2} - \frac{1}{4})}\Gamma(s + \frac{k}{2} - \frac{1}{4}) M_1(s) = \pi^{-(s+\frac{k}{2} + \frac{3}{4})}\Gamma(-s + \frac{k}{2} - \frac{3}{4}) M_2(1-s). \quad (5)$$

We next recall a theorem due to Chandrasekharan and Narasimhan. We state a simplified version of their result for our purpose. We need to define a few things before proceeding to the theorem.

Let

$$\phi(s) = \sum_{n \geq 1} \frac{a(n)}{n^s},$$

and

$$\psi(s) = \sum_{n \geq 1} \frac{b(n)}{n^s}$$

be two Dirichlet series. Let $\Delta(s) = \prod_{i=1}^l \Gamma(\alpha_is + \beta_i)$ and $A = \sum_{i=1}^l \alpha_i$. Assume that

$$Q(x) = \frac{1}{2\pi i} \int_C \frac{\phi(s)}{s} x^s ds,$$

where $C$ encloses all the singularities of the integrand. With these definitions, we now state the theorem [2, Theorem 4.1].

**Theorem 2.2.** Suppose that the functional equation

$$\Delta(s)\phi(s) = \Delta(\delta - s)\psi(\delta - s)$$

is satisfied with $\delta > 0$, and that the only singularities of the function $\phi$ are poles. Then, we have

$$A(x) - Q(x) = O(x^{\frac{k}{2} + \frac{1}{2} + 2a\eta}) + O(x^{\delta - \frac{k}{2} - \eta}(\log x)^{r-1}) + O\left(\sum_{x < n \leq x'} |a(n)|\right),$$

for every $\eta \geq 0$, where $A(x) = \sum_{n \leq x} a(n), x' = x + O(x^{1 - \frac{k}{2} - \eta}), q$ is the maximum of the real parts of the singularities of $\phi$, $r$ is the maximum order of a pole with real
part \( q \), and \( u = \beta - \frac{1}{2} - \frac{1}{16} \), where \( \beta \) is such that \( \sum_{n=1}^{\infty} |b(n)|n^{-\beta} \) is finite. If, in addition, \( a_n \geq 0 \), then we have

\[
A(x) - Q(x) = O(x^{\frac{1}{2} - \frac{1}{16} + 2\eta u}) + O(x^{\frac{1}{2} - \frac{1}{16} - \eta (\log x)^{-1}}).
\]

Next, we recall the following result by Kohnen and Zagier [8, Corollary 5].

**Lemma 2.3.** Let \( F \) be a normalized eigenform of weight \( 2k \) on \( \text{SL}_2(\mathbb{Z}) \). Then the Dirichlet series

\[
\mathcal{L}_F(s) = \sum_{n=1}^{\infty} L(F, (-1)^k n, k) n^{-s}
\]

is absolutely convergent for \( \Re(s) > 1 \), has a meromorphic continuation to the entire complex plane with the only singularity a simple pole at \( s = 1 \), and satisfies the functional equation

\[
\pi^{-2s} \Gamma(s) \Gamma \left( s + k - \frac{1}{2} \right) \zeta(2s) \mathcal{L}_F(s) = \pi^{-2(1-s)} \Gamma(1-s) \Gamma \left( -s + k + \frac{1}{2} \right) \zeta(2-2s) \mathcal{L}_F(1-s).
\]

The residue of \( \mathcal{L}_F(s) \) at \( s = 1 \) is \( \frac{3(4\pi)^{2k}}{\pi(2k-1)} \langle F, F \rangle \).

### 3. Preliminary Results

We now prove several results which are of independent interest.

**Proposition 3.1.** If the \( a(n) \)s are as in Theorem 1.2, then

\[
\sum_{n \leq x} a(n)^2 = Bx + O(x^{\frac{1}{2} + \epsilon}),
\]

where \( B \) is some positive constant.

**Proof.** For \( \Re(s) > 1 \), let

\[
\zeta(2s) \mathcal{L}_F(s) = \sum_{n=1}^{\infty} \frac{b(n)}{n^s}.
\]

Since \( b(n) (n \geq 1) \) are positive, applying Theorem 2.2 to the sequence \( b(n) \), and using Lemma 2.3, we get

\[
\sum_{n \leq x} b(n) - Q(x) = O(x^{\frac{1}{2} - \frac{1}{16} + 4\eta u}) + O(x^{1 - \frac{1}{16} - \eta}),(7)
\]

for all \( \eta \geq 0 \), where for \( \epsilon > 0 \),

\[
u = 1 + \epsilon - \frac{1}{2} - \frac{1}{8} = \frac{3}{8} + \epsilon.
\]
Since both $\zeta(s)$ and $L_F(s)$ have only simple poles at $s = 1$, the poles of $(2s)L_F(s)$ are at $s = \frac{1}{2}$ and $s = 1$. Thus

$$Q(x) = c_1 x + c_2 x^{\frac{1}{2}}$$

for some constants $c_1$ and $c_2$. Choosing $\eta = \frac{3}{8(4u+1)}$ in (7), we get

$$\sum_{n \leq x} b(n) = c_1 x + O(x^{\frac{3}{5}+\epsilon}).$$

(8)

From (6), we see by M"obius inversion that

$$L(F, (-1)^k n, k) = \sum_{d^2 | n} \mu(d) b(n/d^2).$$

Now applying (8), we get

$$\sum_{n \leq x} \sum_{d^2 | n} \mu(d) b(n/d^2) = \sum_{d^2 \leq x} \mu(d) \sum_{\epsilon \leq x/d^2} b(\epsilon) = \sum_{d^2 \leq x} \mu(d) \left\{ c_1 \frac{x}{d^2} + O((x/d^2)^{\frac{3}{5}+\epsilon}) \right\}.$$ 

Thus

$$\sum_{n \leq x} L(F, (-1)^k n, k) = x \left\{ \frac{6c_1}{\pi^2} + O(1/\sqrt{x}) \right\} + O(x^{\frac{3}{5}}) = \frac{6c_1}{\pi^2} x + O(x^{\frac{3}{5}}).$$

In the above expression, we have used the well-known fact that

$$\sum_{d^2 \leq x} \frac{\mu(d)}{d^2} = \frac{6}{\pi^2} + O(1/\sqrt{x}).$$

Now applying (4), we get

$$\sum_{n \leq x} a(n)^2 = Bx + O(x^{\frac{3}{5}+\epsilon}),$$

where $B$ is a positive constant. This proves the required result.

**Proposition 3.2.** Let $a(n)$ be as in Theorem 1.2. Then for any $\epsilon > 0$, we have

$$\sum_{n \leq x} a(n) \ll x^{\frac{3}{5}+\epsilon}.$$ 

**Proof.** We apply Theorem 2.2 to prove this result. From (5), we see that $\delta = 1$, $A = 1$. By Cauchy’s theorem, $Q(x) = \phi(0)$ is a constant. Applying Theorem 2.2 to the sequence $a(n)$ ($n \geq 1$), we get

$$\sum_{n \leq x} a(n) = O(x^{\frac{1}{5}+2\eta u}) + O \left( \sum_{x < n \leq x' \atop x' \leq x} |a(n)| \right)$$

(9)

for every $\eta \geq 0$. Here $x' = x + O(x^{\frac{1}{2}+\eta})$ and $u = 1 + \epsilon - \frac{1}{2} - \frac{1}{3} = \frac{1}{3} + \epsilon$. 

Now by Cauchy–Schwarz, we have
\[ \sum_{x < n \leq x'} |a(n)| \ll x^{\frac{3}{5} - \frac{2\eta}{x'}} \left( \sum_{x < n \leq x'} |a(n)|^2 \right)^{\frac{1}{2}}. \]

Now applying Proposition 3.1 to the above expression, we get
\[ \sum_{x < n \leq x'} |a(n)| \ll x^{\frac{3}{5} - \frac{2\eta}{x'}} \ll x^{\frac{2}{5} - \frac{2\eta}{x'}}. \]

Substituting the above estimate in (9), we get that
\[ \sum_{n \leq x} a(n) = O(x^{\frac{3}{5} + 2\eta}) + O(x^{\frac{2}{5} - \frac{2\eta}{x'}}). \]

Now choosing \( \eta = \frac{3}{5(4u+1)} \), we obtain
\[ \sum_{n \leq x} a(n) = O(x^{\frac{3}{5} + \epsilon}). \]

4. The Ramanujan Conjecture for Half-Integral Weight Forms

The Ramanujan conjecture for half-integral weight cusp forms is the assertion that for any \( \epsilon > 0 \), \( a(n) = O(n^{\epsilon}) \). This would follow from the Lindelöf hypothesis for \( L(f, D, s) \) in the \( D \)-aspect, which is an unsolved problem. However, using the estimate given in the above proposition, we deduce that \( a(n) = O(n^{\frac{3}{5} + \epsilon}) \). This is weaker than a result by Iwaniec [4], who showed that we can take \( \epsilon = \frac{3}{15} \).

Using Proposition 3.1, we prove the following theorem, which is of independent interest.

**Theorem 4.1.** Let \( a(n) \) be as in Theorem 1.2. Fix \( \epsilon > 0 \). Then for any \( \theta > 3/5 \),
\[ \# \{x < n \leq x + x^\theta : |a(n)| > n^\epsilon \} \ll x^{\theta - 2\epsilon}. \]
In particular, there are infinitely many square-free \( n \) with \( (-1)^k n \equiv 0, 1 \pmod{4} \) for which the sequence \( a(n) \) satisfies Ramanujan conjecture.

**Proof.** From the above proposition, for any \( \theta > 0 \),
\[ \sum_{x < n \leq x + x^\theta} a(n)^2 = Bx^\theta + O(x^{\frac{3}{5} + \epsilon}) \ll x^{\max\{\theta, \frac{3}{5} + \epsilon\}}. \]
Thus
\[ x^{2\epsilon} \cdot \# \{x < n \leq x + x^\theta : |a(n)| > n^\epsilon \} \ll \sum_{x < n \leq x + x^\theta} a(n)^2 \ll x^{\max\{\theta, \frac{3}{5} + \epsilon\}}, \]
which gives
\[ \# \{x < n \leq x + x^\theta : |a(n)| > n^\epsilon \} \ll x^{\max\{\theta, \frac{3}{5} + \epsilon\} - 2\epsilon}. \]
In particular, if \( \theta > 3/5 \),
\[
\# \{ x \leq n \leq x + x^\theta : |a(n)| \leq n^\epsilon \} = x^\theta + O(x^{\theta - 2\epsilon}).
\] (10)

We know the well-known fact that for \( \theta > 1/2 \), we have (see [11, Exercise 1.4.4])
\[
\# \{ x \leq n \leq x + x^\theta : n \text{ is square-free} \} = \frac{6}{\pi^2} x^\theta + O(\sqrt{x}).
\]

Thus
\[
\# \{ x \leq n \leq x + x^\theta : n \text{ is not square-free} \} = \left( 1 - \frac{6}{\pi^2} \right) x^\theta + O(\sqrt{x}).
\] (11)

Comparing (10) and (11), we conclude that there is a square-free integer \( n \) in the interval \((x, x + x^\theta]\) such that
\[
|a(n)| \leq n^\epsilon.
\]
This proves the second assertion.

As Kohnen remarks (in a private communication), for any fixed \( t \), we have
\[ a(tn^2) = O(n^\epsilon |a(t)|) \]
by virtue of the Shimura correspondence and the Ramanujan estimate for integral weight forms. However, the \( O \)-estimate depends on \( t \). And so, if \( t \ll n' \), the sequence \( a(tn^2) \) satisfies the Ramanujan estimate. Now, the number of such numbers \( tn^2 \) in the interval is easily seen to be
\[
\ll \sum_{t \leq x^\epsilon} \sqrt{x/t} \ll x^{\frac{1}{2} + \epsilon} = o(x^\theta).
\]
In other words, such numbers form a sparse subset of the set \([x, x + x^\theta]\).

5. Proof of Theorem 1.1

Assume on the contrary that \( a(n) \) are of same sign for all integers \( n \) in the interval \((x, x + x^\epsilon]\). Without any loss of generality we may assume that \( a(n) \) are positive for \( n \in (x, x + x^\epsilon] \). Then we have
\[
\sum_{x < n \leq x + x^\epsilon} a(n)^2 \ll x^\alpha \sum_{x < n \leq x + x^\epsilon} a(n) \ll x^{\alpha + \beta}.
\] (12)

Since \( r > \gamma \), we see from the hypothesis that
\[
x^\epsilon \ll \sum_{x < n \leq x + x^\epsilon} a(n)^2.
\] (13)

Since \( r > \alpha + \beta \), (12) and (13) contradict each other. Thus, there is at least one sign change of the sequence \( a(n) (n \geq 1) \) for \( n \in (x, x + x^\epsilon] \).

6. Proof of Theorem 1.2

We have the Iwaniec’s bound [4] for the Fourier coefficients of half-integral weight cusp forms
\[ a(n) = O(n^{\frac{\gamma}{2}}). \]
From Propositions 3.1 and 3.2, we have
\[ \sum_{n \leq x} a(n)^2 = Bx + O(x^{3/5}) , \]
and
\[ \sum_{n \leq x} a(n) = O(x^{3/5}) . \]
Thus all the required conditions of Theorem 1.1 are satisfied. Hence by Theorem 1.1, there is at least one sign change of the sequence \( a(n)(n \geq 1) \) for \( n \in (x, x + x^\delta] \), where
\[ \delta > \max \left\{ \frac{3}{14} + \frac{2}{5} + \epsilon, \frac{3}{5} + \epsilon \right\} = \frac{43}{70} + \epsilon , \]
for any \( \epsilon > 0 \). This proves that \( \delta > \frac{43}{70} \). If we assume the Ramanujan conjecture, that is,
\[ a(n) = O(n^\epsilon) , \]
for any \( \epsilon > 0 \), from (9), we have
\[ \sum_{n \leq x} a(n) = O(x^{1/4 + 2\eta u} + O(x^{1/4 - \eta + \epsilon})) , \]
for every \( \eta > 0 \), where \( u = \frac{1}{4} + \epsilon \). Choosing \( \eta = \frac{1}{2n+1}(\frac{1}{4} + \epsilon) \) in the above expression, we deduce that
\[ \sum_{n \leq x} a(n) = O(x^{1/4 + \epsilon}) , \]
for any \( \epsilon > 0 \). Then as above,
\[ \delta > \max \left\{ \frac{1}{3} + \epsilon + \epsilon, \frac{3}{5} + \epsilon \right\} = \frac{3}{5} + \epsilon . \]
Since the above inequality is true for any \( \epsilon > 0 \), this proves the last assertion.

7. Concluding Remarks

The method applied here also applies to sequences of complex-valued coefficients. Indeed, in Theorem 1.1, we can deduce that in the interval stated, there is at least one “angular change”. From this, one can deduce a lower bound for the number of “angular changes” in the sequence of complex numbers \( a(n) \), \( n \leq x \) by a suitable modification of our proof.

One can further investigate if our results here are optimal. This is unlikely. Even assuming Ramanujan conjecture, we deduce only \( \gg x^{\frac{3}{5}} \) sign changes for \( n \leq x \). This falls short of the Bruinier–Kohnen conjecture. However, based on analogies with the classical Dirichlet divisor problem and the circle problem, it seems reasonable to
conjecture that
\[ \sum_{n \leq x} a(n) = O(x^{\frac{1}{4} + \epsilon}), \]
and
\[ \sum_{n \leq x} a(n)^2 = Ax + O(x^{\frac{3}{8} + \epsilon}). \]

This is consistent with the philosophy outlined in [10]. Our conjectures lead to
\[ x^{\frac{1}{5} - \epsilon} \] sign changes of \( a(n) \) with \( n \leq x \). (We take this opportunity to correct two
typos in [10]. On p. 525, line 3, \((2A - 1)(2A + 1)\) should be \((2A - 1)/(2A + 1)\) and
on p. 532, line 1, the formula for \( p \) should be \( p = 2 \sum_{j=1}^{N} \alpha_j \).)

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