

Vector Calculus, tutorial 5

October 2013

1. For the function $f(x, y) = 2x^2 + y^2$
 - a) Compute the family of flow lines for the gradient field $\vec{F}(x, y) = -\nabla f(x, y)$.
 - b) Sketch some of the level curves of the function $f(x, y)$ together with some of the lines of steepest descent for this function.

2. For the vector field $\vec{F} = x^2\mathbf{i} + xy\mathbf{j} + 2z\mathbf{k}$, and the curve which is the intersection of the cylinder $(x - 1)^2 + y^2 = 2$ with the plane $x + y + z = 4$, calculate the work done in moving a particle one complete cycle of the curve, with counterclockwise orientation (as viewed from above the plane).

3. Consider the vector field $\vec{F} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.
 - a) Show that $\vec{F}$ is a gradient field.
 - b) Describe the equipotential surfaces of $\vec{F}$ in words and with sketches
 - c) Calculate the work done by the vector field $\vec{F}$ in moving a particle along the parameterized path $\vec{r}(t) = (2, -5, 7) + t(-2, -4, 6)$, $-1 \leq t \leq 1$.