Math 221 Queen's University, Department of Mathematics

Vector Calculus, tutorial 6

October 2013

1. For the parameterized helix $\mathcal{C}$, given by $\vec{r}(t) = \cos(t)\vec{\mathbf{i}} + \sin(t)\vec{\mathbf{j}} + t\vec{\mathbf{k}}$, on the time interval $0 \leq t \leq 1.25\pi$, calculate the path integral

$$\int_{\mathcal{C}} yz^2 e^{xyz^2} dx + xz^2 e^{xyz^2} dy + 2xyze^{xyz^2} dz.$$

2. Consider the vector field $\vec{F} : \mathbb{R} \times (0, +\infty) \to \mathbb{R}^2$ given by

$$\vec{F}(x, y) = \frac{x + xy^2}{y^2}\vec{\mathbf{i}} - \frac{x^2 + 1}{y^3}\vec{\mathbf{j}}.$$

a) Determine whether $\vec{F}$ is a gradient field or not, and give an explanation of your conclusion.

b) Calculate the work done in moving a particle along the curve $y = 1 + x - x^2$ from $(0, 1)$ to $(1, 1)$.

3)Let $\vec{F} = (3x^2y + y^3 + e^x)\,\vec{\mathbf{i}} + \left(e^{y^2} + 12x\right)\vec{\mathbf{j}}$. Consider the line integral of $\vec{F}$ around the circle of radius a, centered at the origin and oriented counterclockwise.

a) Find the line integral for a=1.

b) For which value of a is the line integral a maximum. Give a clear explanation of your conclusion.