Math 221 Queen's University, Department of Mathematics

Vector Calculus, tutorial 7

November 2013

1. The electric field $\vec{E}$, at the point with position vector $\vec{r}$ in $\mathbb{R}^3$, due to a charge q at the origin is given by

$$\vec{E}(\vec{r}) = q\frac{\vec{r}}{\|\vec{r}\|^3},$$

a) Compute curl $\vec{E}$. Is $\vec{E}$ a path independent vector field? Give a clear explanation of your conclusion.

b) If possible, find a potential function for $\vec{E}$.

2. Calculate the area of the bounded region inside the folium of Descartes, $x^3 + y^3 = 3xy$.

a)Sketch the bounded region and show that this region has a boundary which is parameterized by the vector function $\vec{r}(t) : [0, \infty) \to \mathbb{R}^2$

$$\vec{r}(t) = \frac{3t}{1+t^3}\vec{\mathbf{i}} + \frac{3t^2}{1+t^3}\vec{\mathbf{j}}$$

b) Using this parameterization and Green's Theorem calculate the area of the bounded region.

3) The donut shaped surface **S** (called a torus)

$$\left(\sqrt{x^2+y^2} - a\right)^2 + z^2 = b^2, \quad a > b > 0$$

can be parameterized by $\mathbf{T} : [0, 2\pi] \times [0, 2\pi] \to \mathbb{R}^3$.

$$\mathbf{T}(\theta, \phi) = (a + b\cos(\theta))\cos(\phi)\vec{\mathbf{i}} + (a + b\cos(\theta))\sin(\phi)\vec{\mathbf{j}} + b\sin(\theta)\vec{\mathbf{k}}$$

Find the surface area of the torus $\mathbf{S}$.

b) Calculate the area of the ellipse $\mathbf{E}$ on the plane $2x + y + z = 2$ cut out by the circular cylinder $x^2 + y^2 = 2x$.