Math 221 Queen's University, Department of Mathematics

Vector Calculus, tutorial 8

November 2013

1.(a) Find a parameterization of the hyperboloid $x^2 + y^2 - z^2 = 25$.

(b) Find an expression for the outward pointing unit normal on the surface of the hyperboloid.

(c) Find an equation of the tangent plane to the hyperboloid at the point (a,b,0) where $a^2 + b^2 = 25$.

(d) Show that the lines $t \mapsto (a - bt, b + ta, 5t)$ and $t \mapsto (a + tb, b - ta, 5t)$ lie in the surface and also in the tangent plane found in part (c).

2. Let $\vec{H} = (e^{xy} + 3z + 5)\,\vec{\mathbf{i}} + (e^{xy} + 5z + 3)\,\vec{\mathbf{j}} + (e^{xy} + 3z)\,\vec{\mathbf{k}}$. Calculate the flux of the vector field $\vec{H}$ through the square of side length 2 with one vertex at the origin, one edge along the positive y-axis, one edge in the x-z plane with $x > 0, z > 0$ and normal direction $\vec{\mathbf{n}} = \vec{\mathbf{i}} - \vec{\mathbf{k}}$.

3) The donut shaped surface **S** (called a torus)

$$\left(\sqrt{x^2 + y^2} - a\right)^2 + z^2 = b^2, \quad a > b > 0$$

can be parameterized by $\mathbf{T} : [0, 2\pi] \times [0, 2\pi] \to \mathbb{R}^3$.

$$\mathbf{T}(\theta, \phi) = (a + b\cos(\theta))\cos(\phi)\vec{\mathbf{i}} + (a + b\cos(\theta))\sin(\phi)\vec{\mathbf{j}} + b\sin(\theta)\vec{\mathbf{k}}$$

Find the surface area of the torus **S**.

b) Calculate the area of the ellipse **E** on the plane $2x + y + z = 2$ cut out by the circular cylinder $x^2 + y^2 = 2x$.