Math 221 Queen's University, Department of Mathematics

Vector Calculus, tutorial 9

November 2013

1.(a) Evaluate the surface integral $\int\int_{\mathbf{Q}} \vec{\mathbf{E}} \cdot \mathbf{d}\vec{\mathbf{S}}$ for the vector field $\vec{E} = ze^{x^2}\vec{\mathbf{i}} + 3y\vec{\mathbf{j}} + (2 - yz^7)\,\vec{\mathbf{k}}$, and$\mathbf{Q}$ denotes the PWS surface which is the union of the five **upper faces** of the unit cube $[0,1] \times [0,1] \times [0,1]$ (excluding the face with z=0). Orient the surface Q with outward pointing normal $\vec{\mathbf{n}}$.

2. Let S denote the surface defined by $z = e^{1-x^2-y^2}$ with $z \geq 1$, and oriented upwards. Let $\vec{\mathbf{H}} = x\vec{\mathbf{i}} + y\vec{\mathbf{j}} + (2-2z)\,\vec{\mathbf{k}}$. Calculate the flux of the vector field $\vec{\mathbf{H}}$ through the surface S.

3) Consider a vector field $\vec{\mathbf{F}} = \frac{x^3}{3}\vec{\mathbf{i}} + \frac{y^3}{3}\vec{\mathbf{j}} + (3z)\,\vec{\mathbf{k}}$, and a solid region $M \subset \mathbb{R}^3$ with complete closed boundary ∂M. How should we orient the surface ∂M in order to guarantee that $\int_{\partial M} \vec{\mathbf{F}} \cdot \mathbf{d}\vec{\mathbf{S}}$ is negative?

b) Find the net outward flux of the vector field $\vec{\mathbf{G}} = xy\vec{\mathbf{i}} + yz\vec{\mathbf{j}} + zx\vec{\mathbf{k}}$ from a sphere of radius 1, centered at (0,0,0).