HAND IN
answers recorded
on question paper

QUEEN'S UNIVERSITY
FACULTY OF ENGINEERING & APPLIED SCIENCE
DEPARTMENT OF MATHEMATICS & STATISTICS
MTHE 227
DECEMBER 2012
Instructor M. POPA

Student Number: ______________________

- This examination is *three hours* in length.
- The Casio 991 model calculator (or any calculator with a gold or blue sticker) is permitted.
- Data sheets, or other aids are **not** permitted.
- Answers will be evaluated based on their clarity and correctness. Please show all work.
- Answers are to be recorded on the question paper. If necessary, use the back of a page and advise the marker where to look.
- Proctors are unable to respond to queries about the interpretations of exam questions. Do your best to answer exam questions as written.

1	2	3	4	5	6	7	8	total

1. Problem 1. Multiple Choice (10 questions, 3 marks each)

Each question has four possible answers, labeled (A), (B), (C), and (D). Choose the most appropriate answer. Write your answer in the space provided. Write clearly using **UPPERCASE** letters. Illegible answers will be given a zero. You DO NOT need to justify your answer.

(1) Convert the following integral from spherical coordinates to rectangular (Cartesian) coordinates

$$\int_0^{\pi}\int_0^{2\pi}\int_0^{2} 2^{r+1} r^2 \sin\phi \; dr\; d\theta\; d\phi$$

(A) $\displaystyle\int_{-2}^{2}\int_{-\sqrt{4-z^2}}^{\sqrt{4-z^2}}\int_{-\sqrt{4-y^2-z^2}}^{\sqrt{4-y^2-z^2}} 2^{1+\sqrt{x^2+y^2+z^2}}(x^2+y^2+z^2)\, dx\; dy\; dz$

(B) $\displaystyle\int_{-2}^{2}\int_{-\sqrt{4-z^2}}^{\sqrt{4-z^2}}\int_{-\sqrt{4-y^2-z^2}}^{\sqrt{4-y^2-z^2}} 2^{1+\sqrt{x^2+y^2+z^2}}\, dx\; dy\; dz$

(C) $\displaystyle\int_{0}^{2}\int_{0}^{\sqrt{4-z^2}}\int_{0}^{\sqrt{4-y^2-z^2}} 2^{1+\sqrt{x^2+y^2+z^2}}\, dx\; dy\; dz$

(D) none of the above

ANSWER TO (1): ______

(2) Find the tangent unit vector at $(0,1,0)$ to the space curve

$$\vec{r}(t) = t\,\vec{i} + e^t\,\vec{j} - 3t^2\,\vec{k}$$

where $t \in [-2, 2]$.

(A) $\frac{1}{\sqrt{3}}\vec{i} + \frac{1}{\sqrt{3}}\vec{j} + \frac{1}{\sqrt{3}}\vec{k}$

(B) $\frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$

(C) $\dfrac{\vec{i} + e^t\,\vec{j} - 6t\,\vec{k}}{\sqrt{1 + e^{2t} + 36t^2}}$

(D) none of the above

ANSWER TO (2): ______

(3) Find the unit vector with positive z component which is normal to the surface $z = xy + xy^2$ at the point $(1, 1, 2)$.

(A) $\langle -\frac{2}{\sqrt{14}}, -\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}} \rangle$

(B) $\langle 2, 3, -1 \rangle$

(C) $\langle -\frac{2}{\sqrt{11}}, -\frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}} \rangle$

(D) none of the above

ANSWER TO (3): ______

(4) Find **curl**$\overrightarrow{F}$ for the vector field

$$\overrightarrow{F}(x,y,z) = 3x^2\overrightarrow{i} + 2z\overrightarrow{j} - x\overrightarrow{k}.$$

(A) $5\overrightarrow{i} - 4\overrightarrow{j} + \overrightarrow{k}$

(B) $-2\overrightarrow{i} + \overrightarrow{j}$

(C) $6x$

(D) none of the above

ANSWER TO (4): ______

(5) Find $\mathbf{div}\overrightarrow{F}$ for the vector field

$$\overrightarrow{F}(x,y,z) = (\cos x)\overrightarrow{i} + (2\sin y)\overrightarrow{j} - x\overrightarrow{k}.$$

(A) $(-\sin x)\overrightarrow{i} + (2\cos y)j$

(B) $2\cos y - \sin x$

(C) 0

(D) none of the above

ANSWER TO (5): ______

(6) Which of the following vector fields is conservative?

(A) $\overrightarrow{F}(x,y) = (x^2 - yx)\overrightarrow{i} + (y^2 - xy)\overrightarrow{j}$

(B) $\overrightarrow{F}(x,y) = (2e^{xy} + x^5 e^{xy})\overrightarrow{i} + (x^3 e^{xy} + 2y)\overrightarrow{j}$

(C) $\overrightarrow{F}(x,y) = (2xe^{xy} + x^2 y e^{xy})\overrightarrow{i} + (x^3 e^{xy} + 2y)\overrightarrow{j}$

(D) none of the above

ANSWER TO (6): ______

(7) Let $\mathcal{C}$ be a positively oriented, piecewise-smooth, simple closed curve in the plane and let D be the region bounded by $\mathcal{C}$. Then the area of D equals:

(A) $\int_{\mathcal{C}} \frac{1}{2}xdy - \frac{1}{2}ydx$

(B) $\int_{\mathcal{C}} ydx$

(C) $\int_{\mathcal{C}} \frac{1}{2}xdy + \frac{1}{2}ydx$

(D) none of the above

ANSWER TO (7): ______

(8) If C is the curve parametrized by $\vec{r}(t) = (3\sin t)\,\vec{i} + (3\cos t)\,\vec{j}$ for $t \in [0, 2\pi]$ and

$$\vec{F} = (y - x)\,\vec{i} + (x - 2)\,\vec{j},$$

then the integral $\int_C \vec{F} \cdot d\vec{r}$ equals

(A) 12π

(B) 22

(C) 0

(D) none of the above

ANSWER TO (8): ______

(9) Let E be a simple solid region and let S be the boundary surface of E, with positive outward orientation. Then the volume of E equals:

(A) $\iint_S x\,\vec{i} \cdot d\vec{S}$

(B) $\iint_S (x\,\vec{i} + y\,\vec{j}) \cdot d\vec{S}$

(C) $\iint_S (x\,\vec{i} + y\,\vec{j} + z\,\vec{k}) \cdot d\vec{S}$

(D) none of the above

ANSWER TO (9): ______

(10) Evaluate

$$\iint_S \vec{F} \cdot d\vec{S}$$

where $\vec{F}(x,y,z) = y^2\,\vec{i} + xz^2\,\vec{j} + y^3\,\vec{k}$ and S is the surface given by $x^4 + y^4 + z^2 = 16$.

(A) $12\pi^2$

(B) 0

(C) $\frac{16}{3}\pi$

(D) none of the above

ANSWER TO (10): ______

Problem 2.
Evaluate

$$\iiint_E (x+3)dV$$

where E is the solid given by the relations

$$\begin{aligned} 0 &\leqslant x+y-z \leqslant 4 \\ -1 &\leqslant 2z-x-2y \leqslant 2 \\ 2 &\leqslant \frac{1}{3}(2z-x-y) \leqslant 3. \end{aligned}$$

Problem 3.
Find the area of the surface $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

Problem 4.
Use Green's Theorem to evaluate the line integral

$$\int_C (y + e^x)dx + (2x + \cos y)dy$$

where C is the triangle with vertices $(0,0)$, $(0,5)$ and $(5,5)$ oriented counterclockwise.

Problem 5.

Use Stoke's Theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$ where

$$\vec{F}(x,y,z) = yz\,\vec{i} + 3xz\,\vec{j} + e^{xy}\,\vec{k}$$

and C is the circle $x^2 + y^2 = 16$, $z = 5$, oriented counterclockwise as viewed from above.

Problem 6.

(1) For $f(x, y, z) = x^3 + xyz + e^z$, evaluate ∇f.

(2) Evaluate $\int_C (3x^2 + yz)dx + xzdy + (xy + e^z + 2)dz$, where C is the line segment from $(0, 0, 0)$ to $(1, 1, 1)$.

Problem 7.
Use the Divergence Theorem to compute the integral

$$\iint_S \vec{F} \cdot d\vec{S}$$

where

$$\vec{F}(x,y,z) = (3z^2 + 2x)\,\vec{i} + (5y + x^2)\,\vec{j} + (xy^3 + 4)\,\vec{k}$$

and S is the surface of the sphere of radius 1 and center $(1,1,1)$, oriented outwards.

Problem 8. Evaluate the triple integral

$$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-\sqrt{x^2+y^2}}^{\sqrt{x^2+y^2}} xz \, dz \, dy \, dx$$