

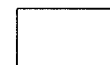
QUEEN'S UNIVERSITY
FACULTY OF ENGINEERING & APPLIED SCIENCE
DEPARTMENT OF MATHEMATICS & STATISTICS
MTHE 227
DECEMBER 2012
Instructor M. POPA

Student Number: _____

- This examination is *three hours* in length.
- The Casio 991 model calculator (or any calculator with a gold or blue sticker) is permitted.
- Data sheets, or other aids are **not** permitted.
- Answers will be evaluated based on their clarity and correctness.
Please show all work.
- Answers are to be recorded on the question paper. If necessary, use the back of a page and advise the marker where to look.
- Proctors are unable to respond to queries about the interpretations of exam questions. Do your best to answer exam questions as written.

1	2	3	4	5	6	7	8	total

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**1. Problem 1.** Multiple Choice (10 questions, 3 marks each)

Each question has four possible answers, labeled (A), (B), (C), and (D). Choose the most appropriate answer. Write your answer in the space provided. Write clearly using **UPPERCASE** letters. Illegible answers will be given a zero. You **DO NOT** need to justify your answer.

(1) Convert the following integral from spherical coordinates to rectangular (Cartesian) coordinates

$$\int_0^\pi \int_0^{2\pi} \int_0^2 2^{r+1} r^2 \sin \phi \, dr \, d\theta \, d\phi$$

- (A) $\int_{-2}^2 \int_{-\sqrt{4-z^2}}^{\sqrt{4-z^2}} \int_{-\sqrt{4-y^2-z^2}}^{\sqrt{4-y^2-z^2}} 2^{1+\sqrt{x^2+y^2+z^2}} (x^2 + y^2 + z^2) \, dx \, dy \, dz$
- (B) $\int_{-2}^2 \int_{-\sqrt{4-z^2}}^{\sqrt{4-z^2}} \int_{-\sqrt{4-y^2-z^2}}^{\sqrt{4-y^2-z^2}} 2^{1+\sqrt{x^2+y^2+z^2}} \, dx \, dy \, dz$
- (C) $\int_0^2 \int_0^{\sqrt{4-z^2}} \int_0^{\sqrt{4-y^2-z^2}} 2^{1+\sqrt{x^2+y^2+z^2}} \, dx \, dy \, dz$
- (D) none of the above

ANSWER TO (1): _____



(2) Find the tangent unit vector at $(0, 1, 0)$ to the space curve

$$\vec{r}(t) = t\vec{i} + e^t\vec{j} - 3t^2\vec{k}$$

where $t \in [-2, 2]$.

- (A) $\frac{1}{\sqrt{3}}\vec{i} + \frac{1}{\sqrt{3}}\vec{j} + \frac{1}{\sqrt{3}}\vec{k}$
(B) $\frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$
(C) $\frac{\vec{i} + e^t\vec{j} - 6t\vec{k}}{\sqrt{1 + e^{2t} + 36t^2}}$
(D) none of the above

ANSWER TO (2): _____



(3) Find the unit vector with positive z component which is normal to the surface $z = xy + xy^2$ at the point $(1, 1, 2)$.

(A) $\langle -\frac{2}{\sqrt{14}}, -\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}} \rangle$

(B) $\langle 2, 3, -1 \rangle$

(C) $\langle -\frac{2}{\sqrt{11}}, -\frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}} \rangle$

(D) none of the above

ANSWER TO (3): _____



(4) Find $\text{curl } \vec{F}$ for the vector field

$$\vec{F}(x, y, z) = 3x^2 \vec{i} + 2z \vec{j} - x \vec{k}.$$

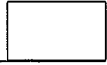
(A) $5 \vec{i} - 4 \vec{j} + \vec{k}$

(B) $-2 \vec{i} + \vec{j}$

(C) $6x$

(D) none of the above

ANSWER TO (4): _____



(5) Find $\text{div } \vec{F}$ for the vector field

$$\vec{F}(x, y, z) = (\cos x) \vec{i} + (2 \sin y) \vec{j} - x \vec{k}.$$

(A) $(-\sin x) \vec{i} + (2 \cos y) \vec{j}$

(B) $2 \cos y - \sin x$

(C) 0

(D) none of the above

ANSWER TO (5): _____



(6) Which of the following vector fields is conservative?

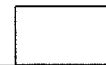
(A) $\vec{F}(x, y) = (x^2 - yx)\vec{i} + (y^2 - xy)\vec{j}$

(B) $\vec{F}(x, y) = (2e^{xy} + x^5e^{xy})\vec{i} + (x^3e^{xy} + 2y)\vec{j}$

(C) $\vec{F}(x, y) = (2xe^{xy} + x^2ye^{xy})\vec{i} + (x^3e^{xy} + 2y)\vec{j}$

(D) none of the above

ANSWER TO (6): _____



(7) Let C be a positively oriented, piecewise-smooth, simple closed curve in the plane and let D be the region bounded by C . Then the area of D equals:

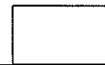
(A) $\int_C \frac{1}{2}x dy - \frac{1}{2}y dx$

(B) $\int_C y dx$

(C) $\int_C \frac{1}{2}x dy + \frac{1}{2}y dx$

(D) none of the above

ANSWER TO (7): _____



(8) If C is the curve parametrized by $\vec{r}(t) = (3 \sin t) \vec{i} + (3 \cos t) \vec{j}$ for $t \in [0, 2\pi]$ and

$$\vec{F} = (y - x) \vec{i} + (x - 2) \vec{j},$$

then the integral $\int_C \vec{F} \cdot d\vec{r}$ equals

- (A) 12π
- (B) 22
- (C) 0
- (D) none of the above

ANSWER TO (8): _____



(9) Let E be a simple solid region and let S be the boundary surface of E , with positive outward orientation. Then the volume of E equals:

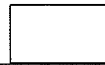
(A) $\iint_S x \vec{i} \cdot d\vec{S}$

(B) $\iint_S (x \vec{i} + y \vec{j}) \cdot d\vec{S}$

(C) $\iint_S (x \vec{i} + y \vec{j} + z \vec{k}) \cdot d\vec{S}$

(D) none of the above

ANSWER TO (9): _____



(10) Evaluate

$$\iint_S \vec{F} \cdot d\vec{S}$$

where $\vec{F}(x, y, z) = y^2\vec{i} + xz^2\vec{j} + y^3\vec{k}$ and S is the surface given by $x^4 + y^4 + z^2 = 16$.

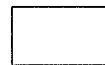
(A) $12\pi^2$

(B) 0

(C) $\frac{16}{3}\pi$

(D) none of the above

ANSWER TO (10): _____

**Problem 2.**

Evaluate

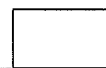
$$\iiint_E (x + 3) dV$$

where E is the solid given by the relations

$$0 \leq x + y - z \leq 4$$

$$-1 \leq 2z - x - 2y \leq 2$$

$$2 \leq \frac{1}{3}(2z - x - y) \leq 3.$$



Problem 3.

Find the area of the surface $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

**Problem 4.**

Use Green's Theorem to evaluate the line integral

$$\int_C (y + e^x)dx + (2x + \cos y)dy$$

where C is the triangle with vertices $(0, 0)$, $(0, 5)$ and $(5, 5)$ oriented counterclockwise.

**Problem 5.**

Use Stoke's Theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$ where

$$\vec{F}(x, y, z) = yz \vec{i} + 3xz \vec{j} + e^{xy} \vec{k}$$

and C is the circle $x^2 + y^2 = 16$, $z = 5$, oriented counterclockwise as viewed from above.

**Problem 6.**

(1) For $f(x, y, z) = x^3 + xyz + e^z$, evaluate ∇f .

(2) Evaluate $\int_C (3x^2 + yz)dx + xzdy + (xy + e^z + 2)dz$, where C is the line segment from $(0, 0, 0)$ to $(1, 1, 1)$.

**Problem 7.**

Use the Divergence Theorem to compute the integral

$$\iint_S \vec{F} \cdot d\vec{S}$$

where

$$\vec{F}(x, y, z) = (3z^2 + 2x) \vec{i} + (5y + x^2) \vec{j} + (xy^3 + 4) \vec{k}$$

and S is the surface of the sphere of radius 1 and center $(1, 1, 1)$, oriented outwards.



Problem 8. Evaluate the triple integral

$$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-\sqrt{x^2+y^2}}^{\sqrt{x^2+y^2}} xz \, dz \, dy \, dx$$